Statistics on mathematics achievement show that Latinos are significantly under-represented in all scientific and engineering careers in direct proportion to the amount of mathematics required for a particular job (Secada 1992; National Research Council 1989). A factor in the low levels of achievement among Latino students may be the misconception among educators that since mathematics uses symbols, it is therefore “culture free” and ideal for instructing students who are still learning the English language, or “English-language learners” (California Department of Education 1990). This misconception ignores the vital role of language in the development of mathematical concepts. Mathematics power is rooted in a strong conceptual understanding of mathematics, and this conceptual base is best developed through concrete experiences and language (National Council of Teachers of Mathematics [NCTM] 1989).

The relationship between language proficiency and mathematics achievement has been documented by researchers such as De Avila and Duncan (1981), who found that the low achievement in mathematics of Latino English-language learners (ELL) can be attributed to low levels of English proficiency. A lack of understanding about the role of language in mathematics instruction has led either to unreasonably high expectations for English-language learners’ achievement in situations in which they receive no linguistic support or to lowered expectations that deny equal access to mathematical skills and reasoning (Secada 1992). The dilemma faced by the mathematics teacher of English-language learners is this: How should mathematics be taught to make a meaningful and powerful curriculum accessible to English-language learners?

Concept development occurs naturally when all students are fluent in the language of instruction, whether that be English, Spanish, or another language. However, many classrooms contain students at various levels of English proficiency, and teachers are faced with the challenge of covering new concepts in a language all students can understand. Many instructional strategies aim at making instruction accessible. According to the research in psycholinguistic development and learning, one of the key elements in teaching second-language learners is what Krashen (1981) terms comprehensible input. This is a construct developed to describe language that is understandable and meaningful under optimal conditions. Krashen explains this theory in
instructional terms with the formula $i + 1$, where instruction ($i$), in order to be comprehensible and promote second-language fluency must be one level more complex than the second-language proficiency of the student. This allows for the language to be understood while also challenging the learners to increase their second-language proficiency. Comprehensible input is provided in school contexts through certain planned strategies, including the use of concrete contextual referents and the lack of restriction on the use of the students’ primary language. The principle of $i + 1$ is the basis for what is termed Specially Designed Academic Instruction in English (SDAIE), also called sheltered English instruction, advocated for English-language learners (Díaz-Rico and Weed 1995). Sheltered instruction includes a set of strategies used to adapt content-area material for English-language learners.

The formula $i + 1$ was originally developed to describe the effective instruction of ELL students. The formula refers to instruction that is just one level of linguistic complexity beyond students’ current level of understanding. The principle of $i + 1$ applies across curricular areas, since effective instruction is based on advancing students from their current level of understanding to the next level of complexity. This developmental process is also advocated in mathematics instruction.

The National Council of Teachers of Mathematics (NCTM) advocates an incremental developmental process in mathematics instruction in *Curriculum and Evaluation Standards for School Mathematics* (1989). These standards offer guidance on how to construct a conceptual understanding of mathematics by urging teachers to begin each new concept with concrete examples and experiences. The curriculum should then provide opportunities for students to make connections among concrete experiences, semiconcrete graphical depiction, abstract symbolic representations, verbal language, and written expression to develop a thorough understanding of the new concept.

Martorella (1986) describes concepts as categories into which knowledge and experiences are grouped or classified. This process of classification and integration is ongoing, and the learner is constantly sorting, relating, and extending conceptual categories. Concepts can range from the simple to the very complex and from concrete to abstract.

Concept development can be the primary goal in mathematics classrooms where all students are proficient in the language of instruction. However, in linguistically diverse classrooms, teachers must also consider the linguistic complexity of the language used in instruction and the language proficiency of the students in order to provide comprehensible input. If new concepts are introduced in an unfamiliar language, students must struggle with two unknowns: the language and the concept. This dual challenge of unknowns makes learning formidable (De Avila 1984).

The field of mathematics requires students to think in terms of concepts, abstract ideas, and symbols. Roe, Stoodt, and Burns (1987) describe mathematics as a highly compressed form of communication in which a single symbol may represent several words or relationships. For example, the equals sign (=) represents the concept of equality and the “greater than or equal to” symbol ($\geq$) simultaneously depicts the concepts of relative numerical value and equality.

In addition, terms in mathematics such as *count*, *odd*, or *times* may have a definition that is different from their meaning in everyday conversation. These polysemous terms can cause confusion among English-language learners because the students must discern the specialized meaning of common terms in a mathematical context. Since most English language learners first encounter the word *odd* as meaning unusual or different, when they hear a number referred to as odd, they may assume that something about the number is incorrect. The point of the teacher’s presentation is often lost while students grapple to make sense of both new vocabulary and words with multiple meanings. Mathematics teachers must therefore consider both the linguistic and conceptual complexity of the language of instruction when designing lessons.
The theoretical foundations for English-language instructional models are based on the relationship between the language of instruction and the development of concepts and skills. Cummins (1984) documented research from five different sources that supports the theory that there is a common underlying language proficiency in bilingual students that facilitates a transfer of learning between a student's first and second language.

This transfer of learning takes place when a certain “threshold” of proficiency is attained in the second language and when concepts and cognitive skills are firmly established. The data from studies of immigrant students’ acquisition of academic skills and English and from successful bilingual programs in many countries indicate that although language-minority students develop a relatively high level of English proficiency in basic communication, the ability to understand and perform in academic settings, where language is more abstract and complex, develops at a much slower rate. Most English-language learners are unlikely to attain this proficiency in academic language until the later grades of elementary school (Cummins 1981).

Legarreta-Marcaida (1981) describes the preview-review technique as a strategy in which new learning is initially introduced and explained in the students’ native language and later presented and reinforced in the second language by focusing on the new labels and vocabulary used to describe the previously established concepts. The purpose of this instructional strategy is twofold: (1) to ensure comprehensible input for English-language learners and (2) to establish skills and concepts soundly in order to facilitate the transfer of learning from the first to the second language as students acquire the necessary second-language proficiency.

The basic principle underlying the different strategies for bilingual instruction can be summarized as follows: To teach an unknown concept, use the known language; to teach an unknown language, use a known concept. The relationship between language and concept development to ensure comprehensible input for instruction is represented in Table 5.1.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Language</th>
<th>Concept</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Unknown</td>
<td>Unknown</td>
<td>Limited learning opportunity; modify instruction</td>
</tr>
<tr>
<td>B</td>
<td>Known</td>
<td>Unknown</td>
<td>Concept development</td>
</tr>
<tr>
<td>C</td>
<td>Unknown</td>
<td>Known</td>
<td>Language development</td>
</tr>
<tr>
<td>D</td>
<td>Known</td>
<td>Known</td>
<td>Concept and language mastery, advance to next conceptual or linguistic level</td>
</tr>
</tbody>
</table>

Domain A: Unknown Language, Unknown Concept

When a student understands neither the language of instruction nor the concept presented, learning opportunities are limited. For example, when a new concept such as place value is introduced to a student who has little command of the language...
of instruction, the student may confuse numbers such as fourteen and forty or eighteen and eighty because they sound very similar in English. While the student is trying to determine what number the teacher said and to translate it into the primary language, he or she misses the explanation that the 4 represents four groups of ten and 0 indicates that there are no additional units to include. As a result, it is unlikely that the student will either master the concept or advance linguistically through this lesson. On the contrary, he or she may experience frustration that further impedes understanding. The following scenario further illustrates this situation in Domain A.

*Domain A in the Classroom*

Ms. A., who teaches a linguistically diverse classroom of third graders, is starting a unit on multiplication. She is explaining that multiplication is actually repeated addition. Enrique, a monolingual Spanish-speaking student, sits and carefully watches what Ms. A. is doing. Her animated gestures give few clues about the topic of the lecture. After a few minutes, Enrique's attention drifts and he starts playing with his pencil. Ms. A. concludes her instruction and hands out a worksheet for the students to complete. Eager to finish his work, Enrique attempts the first problem but quickly gets frustrated and stops, sits, and waits because he has understood neither the language nor the concept presented.

To revisit the $i + 1$ theory, language is learned most rapidly when presented at a level at which the student understands most of what is being said and needs to determine only a limited number of words from the context. When too much new vocabulary is presented, the student loses the thread of the conversation, and the level of comprehension plummets. For the student to comprehend and learn, instructional modifications must be made.

Students will not persist in trying to make sense of instruction when, despite their best efforts, they cannot comprehend the input. Therefore, instruction in the “unknown language, unknown concept” domain must be modified and adapted so that it is comprehensible to the student. At this juncture, the teacher must decide whether to go through known language to develop concepts (Domain B, see below) or whether to go through known concepts to develop language (Domain C, see below). The decision is based on such factors as the goals of instruction, the possibility of teaching in the students’ primary language, and the ability of the teacher to modify the linguistic and conceptual input in the second language.

*Domain B: Known Language, Unknown Concept*

In bilingual education classrooms where instruction is provided in the primary language, basic concepts are developed in the language familiar to the students. When the language of instruction is removed as an obstacle to learning concepts, teachers can focus instruction on the systemic development of conceptual knowledge. This concept development occurs naturally when all students are fluent in the language of instruction, whether that be English, Spanish, or another language. Teachers planning to work in Domain B must first determine whether students are fluent in the academic language of their primary language. Some students whose primary language is Spanish may be very fluent in conversational Spanish because they learned the language at home and are fully capable of communicating in Spanish. However, fluency in conversational Spanish does not equate to fluency in the underlying vocabulary and concepts of academic Spanish. For example, a lesson covering the concept of using data to make predictions might ask students to perform stock market predictions on the basis of the percentage of growth in certain stocks over a period of time. If all students in the class are fluent Spanish speakers, the teacher might give instructions in Spanish using the term *porcentaje* (percentage). This lesson will be in Domain B (Known Language, Unknown Skill) only for the bilingual students who already understand the underlying concept of percentage. Even those students who recognize the term *porcentaje* may not understand the concept or know how percentages are derived. Therefore, teachers must determine that students understand the underlying concepts of a lesson before a new concept or skill can be successfully introduced through the primary language.
Mathematics has the unique advantage of being able to employ a variety of modalities for instruction. Beyond verbal language, ideas can be expressed through graphical depiction, symbolic representations, and the manipulation of concrete objects. Each of these other modes of presentation has the distinct advantage of being relatively independent of any particular spoken language, which allows new mathematical concepts to be successfully introduced to English-language learners.

If material is presented verbally, English-language learners benefit when the rate of speech is slowed, important terms are defined, and syntactic structures are simplified by using shortened, regularly patterned sentences. Visuals, schematic drawings, and demonstrations also boost the comprehension of verbally presented material. Gonzales (1994) describes these strategies of “sheltered instruction” as a synthesis of the components of quality teaching with the principles of language acquisition. In the sheltered environment, the teacher’s task is to use the principles of second-language learning to build on the background knowledge students have acquired in their native language. The following example illustrates a possible instance of this situation in the classroom.

**Domain B in the Classroom**

Mr. B. teaches in a bilingual classroom where most students receive instruction in Spanish. Before he introduces the concept of multiplication, Mr. B. reviews the foundation concepts and vocabulary in both Spanish and English to ensure that the students understand. The students brainstorm items that come in groups, such as six packs of sodas, five sticks of gum in a package, twelve eggs in a dozen, and so forth. A table of the items is written, in both English and Spanish, on chart paper and then expanded to show how many items are included in one group, two groups, and three groups. Students are placed in teams and complete a similar chart that tells how many eyes, noses, and fingers are represented on their team. As a culminating activity, each student explains in his or her mathematics journal, in either Spanish or English, how the answers to the team chart were determined.

**Domain C: Unknown Language, Known Concept**

Once the conceptual foundation has been established in English-language learners, concepts can be used as a vehicle for learning English. When students are familiar with the topic used in an English-language lesson, they can better anticipate what is going to be said and more readily determine words from the context. Instruction of this type is used in transition and English-as-a-second-language (ESL) classrooms, where the primary goal is to develop skills in English.

Students who already have a strong conceptual base in the subject matter can focus on learning the new vocabulary, phonology, and syntax of their second language to express or explain already familiar concepts. Students who lack a conceptual base, however, often struggle in an immersion program because of the complexities of developing mathematical concepts presented in an unfamiliar language.

Teachers whose students are of mixed English-language abilities can use Domain C (Unknown Language, Known Concept) instruction as a reinforcement and follow-up to Domain B (Known Language, Unknown Concept) instruction. For example, after students have started developing a conceptual understanding of the lesson through instruction in the known or familiar language, the English vocabulary to support the lesson is introduced. Doing so puts the new material at a comprehensible level and focuses on the language-development aspects of learning while solidifying the conceptual learning base. To ensure that students have mastered a concept, mastery should be assessed in the students’ dominant language or language of preference. Portfolios are a valuable tool in assessing the level of understanding of mathematical concepts, especially with English-language learners. (See “Portafolio de Matemática” in this volume.)

Occasionally, instruction is provided in the opposite order by first teaching the supporting English terms and then constructing the concept. This sequencing is not usually recommended by experts in either mathematics or bilingual education, since language and vocabulary are difficult to comprehend and retain without an understanding of
the concepts they represent. Therefore, first developing a concept of place value through the use of manipulatives, graphic depiction, and so forth aids in learning the accompanying vocabulary. This condition is depicted in the following scenario.

**Domain C in the Classroom**

Mr. C. teaches a beginning ESL class to adults at the local community center. His students are recent immigrants who received their basic education in languages other than English. The focus of today’s lesson is to teach the meaning of more, most and fewer, fewest as well as the supporting vocabulary word *cube*. Students arrange their desks in a circle, and each student is given several small cubes. Mr. C. picks up one cube and states, “This is a *cube*. What is this called?” Julia replies, “A *cube*.” Mr. C. picks up two additional cubes and says, “I have three *cubes*. Juan, how many *cubes* do you have?” Juan responds, and several other students also tell the number of cubes they have.

Mr. C. moves to two adjacent students. Norma has five *cubes* and Alicia has three *cubes*. Norma has *more cubes* than Alicia. Pedro has *more cubes* than Ivan. Who has *more cubes* than Tran?"

Comparisons are made between several other pairs of students, and then the students are encouraged to make additional comparisons. A similar format is followed for *most*, *fewer*, and *fewest*. Students are able to infer the meaning of the words through relating the repeated concrete examples to the already established concepts of *cube*, *more*, *most*, *few*, and *fewest*.

**Domain D: Known Language, Known Concept**

The domain of known concepts and known language provides a venue for review and evaluation. When the students have mastered both the language and the concepts presented in the curriculum, the students should be given an opportunity to demonstrate their competence. When assessing students’ learning, special care must be taken to ensure that the student has command of the language used to communicate mastery. If the students do not control the language enough to express their thinking clearly and precisely, the teacher will not be able to discern whether poor performance in mathematics has a conceptual or a linguistic root. To address this problem, a series of evaluation alternatives appropriate for English-language learners is described toward the end of this paper.

**Domain D in the Classroom**

Ms. D. teaches mathematics to a linguistically diverse group of eighth graders. She starts the preparation for an upcoming examination on measurement with a review of the vocabulary used throughout the unit. Images depicting important vocabulary words as well as Spanish translations adorn much of the room. At the conclusion of the vocabulary review, students are given two objects—one a cylinder, the other a rock—and asked to work in groups to determine the volume of each.

The groups quickly start into action, each talking in a mixture of English and Spanish, depending on the language ability of the members. The students practice using mathematical language and review newly learned concepts in measurements as they set about solving the problems at hand. Near the end of the period, Ms. D. asks for the groups to report on their findings to the whole class. The other groups listen, question, challenge, and learn from their peers and Ms. D.’s comments.

Finally, the students are asked to write the definition of *volume*, in either English or Spanish, and give examples of how volume is important in real-life situations. Although most classroom instruction and classwork are completed in English as a measure to increase fluency in English, tests given to measure concept mastery can be completed in English or Spanish.

**HOW TO MAKE INPUT COMPREHENSIBLE**

Language-minority students can understand mathematics instruction if the concepts and operations are presented using techniques that make the concepts comprehensible.
to the students. Appropriate instruction requires additional language and academic support through specialized teaching strategies and attention to the students’ particular linguistic and academic needs (Díaz-Rico and Weed 1995). Teachers need not be fluent in the students’ first language to be successful in teaching English-language learners. However, a knowledge of the basic principles of second-language learning and sound teaching practices, along with a commitment to modifying instructional strategies, will produce increased academic growth in second-language students.

Teaching Vocabulary

The close relationship between vocabulary and concept development was established by researchers working with hearing-impaired students. They discovered that concepts were more difficult to develop and retain in students who have restricted vocabularies (Bracken and Cato 1986). This research further establishes a rationale for why mathematics teachers should fully develop concepts through concrete experiences and systematically teach accompanying vocabulary to assist in the comprehension and retention of concepts in mathematics. Words are labels for thoughts, ideas, concepts, and thinking, and vocabulary is central to concept formation, understanding, and articulation.

Both concepts and vocabulary are acquired and refined gradually (Roe, Stoodt, and Burns 1987). A mathematics teacher cannot assume that English-language learners understand the meaning of words simply because they can pronounce them or use them in coherent sentences. Identifying words with the concepts they label implies both familiarity with the word and the association of ideas with the linguistic form.

As English-language learners make the transition from primary language into English instruction, the English equivalents for the mathematical terms they learned in their primary language might not be covered in the upcoming lessons, creating gaps in their English vocabulary that are irregular and unpredictable. Therefore, mathematics teachers should review or preview all essential vocabulary at the beginning of a lesson or unit, especially when English-language learners are in the class. New mathematical vocabulary in the second language, however, is most effectively introduced after students have established the concepts the vocabulary words represent so that they learn the new “label” for the known concept.

Using Manipulatives

Relating new vocabulary to tangible objects is one of the basic premises of second-language instruction. Words are easier to remember when students can see and touch the objects they represent while repeatedly hearing and saying the new words. Mathematics manipulatives provide excellent opportunities for this type of vocabulary development. A sample list of concepts and the associated vocabulary that can be built using mathematics manipulatives is shown in table 5.2.

The close link between words and mathematical concepts can be seen by the listing shown in table 5.2. Each of the terms represents both an English word and a universal concept. For example, greater than represents the English terminology to describe comparative number or size. Greater than is also a universal concept that is present in all languages. Once the concept of greater than has been established, students do not need to relearn the concept in English; they need only to learn the English label for the previously established concept. Therefore, vocabulary must be learned in each new language, but concepts, once mastered, do not need to be retaught (Cummins 1984).

Building Context

English-language learners communicate more readily when the conversation takes place in a known context. A familiar context helps the listener limit the likely topics of conversation and better anticipate what the speaker is going to say. For example,
English-language learners will find familiar cues to help them anticipate and understand what a clerk is asking in a store or a teller is saying at the bank because in either situation, the conversations tend to follow a predictable pattern.

The same principle applies in mathematics instruction. Context helps students both understand problems and evaluate the reasonableness of their responses. New mathematics curricula use this principle by building lessons around a central theme that is developed over weeks or months. Mathematics problems are extracted from situations that arise as the unit unfolds. This type of curriculum benefits English-language learners because they have time to understand the context of the lessons, relate the context to previous experiences, anticipate the types of problems that might be asked, and develop the necessary vocabulary to comprehend the topics and communicate their thinking. Developing the context also helps all students understand the reasonableness of their responses. Each of these aspects helps English-language learners develop and demonstrate their full mathematics potential.

### Working in Groups

Student work groups provide opportunities for students to develop both listening and speaking skills in English and to increase mathematical understanding. Most learners of a second language are hesitant to speak in front of the whole class; however, they will speak freely in a small group. Small-group settings afford all students greater opportunities to express their ideas, and they give bilingual students important practice in both receptive and expressive English. Student work groups are also a valuable teaching tool, since students who did not fully understand the lesson presented will have access to peer guides and interpreters as they proceed through the assignment.

The interaction in a small group also fosters concept development as students receive input from peers, explain their reasoning, get feedback from others, and refine their thinking on the basis of additional information. This process allows students to internalize the ideas presented and come to a fuller understanding of the concepts taught in the lesson (Kirsner and Bethell 1992).

### Table 5.2

<table>
<thead>
<tr>
<th>Concept</th>
<th>Manipulative</th>
<th>Vocabulary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
<td>Ruler, yardstick</td>
<td>Inch, foot, yard</td>
</tr>
<tr>
<td>Relationships</td>
<td>Tangrams, attribute blocks</td>
<td>Larger, smaller, more, less, greater than, less than, same, different, curve, straight, above, below, next to, big, bigger, biggest</td>
</tr>
<tr>
<td>Critical attributes</td>
<td>Tangrams, attribute blocks</td>
<td>Width, height, length</td>
</tr>
<tr>
<td></td>
<td></td>
<td>circle, triangle, square, color</td>
</tr>
<tr>
<td></td>
<td></td>
<td>edge, side, corner, vertex</td>
</tr>
</tbody>
</table>

For students to demonstrate competence, they should respond about a known concept using a known language. If students lack control of the language, they will not be
able to express their understanding of a concept clearly. The fortunate aspect of mathematics instruction is that a variety of response options, or “languages,” is available, so students who have difficulty expressing their thinking in English can use other modes of communication while they are developing English-language skills. The “languages” available to express mathematical ideas include written responses (in either the primary or the secondary language), graphical representations, and mathematical symbols. By employing a variety of response options, teachers can optimize the opportunity for English-language learners to express their mathematical thinking.

Developing Written Responses

The difficulty of expressing ideas in a secondary language can be mitigated by having students write their responses in the language of greatest fluency. During this writing process, the language difficulty is lessened and the students are able to concentrate on concept and thought development. If the teacher is unable to read the response, the students can translate their writing. During the translation stage, the students need to concentrate only on language, since their thinking has already been recorded. Teachers can also use other students or aides to help with the translation.

An illustration of how students can answer more completely in their primary language can be seen in Janet’s work (see figs. 5.1 and 5.2). After a lesson on determining the area of a triangle, the students in Janet’s class were asked to explain the process in English. Since Janet was a recent immigrant, her English was very limited, and her response provided little information about her understanding of the concept (fig. 5.1). Janet’s teacher asked her to do the problem again, answering in Spanish (fig. 5.2). This response gives a much greater insight into her grasp of the concept.

![Fig. 5.1. Janet’s incomplete response in English gives little information about her understanding of the concept of the area of a triangle.](image)

Writing is the most advanced linguistic skill, the most difficult to learn, and therefore usually the last skill mastered by second-language learners (Díaz-Rico and Weed 1995). Speech provides English-language learners with an easier vehicle to communicate because they can use gestures, voice intonation, and both verbal and nonverbal cues from the listener to aid communication. Mathematics students who have a verbal command of English but do not supply complete written responses should be provided the opportunity to exhibit their mathematical competence orally. They can do so both formally, as in a test, or informally while they are working on a problem in groups.

Writing and speaking are both important tools to help students develop mathematical reasoning, but students who have yet to master these skills in English need to have other response options while their English-language ability is being developed.
Fig. 5.2. Janet's response in Spanish gives a greater insight into her grasp of the concept of the area of a triangle.

Using Graphical Representations

Demonstration

Demonstrating a concept is an important means of communicating mathematical thinking. When teachers tell students, “Show me,” students must use the available manipulatives or tools to demonstrate the concept. These props can support the developing English-language skills of students by decreasing the amount of language needed to communicate effectively. Encouraging students to use manipulatives or other tools to demonstrate their thinking will both stimulate their intellect and facilitate communication.

Graphical Depiction

Many mathematical concepts can be represented more clearly through drawings than through words. The concept of multiplication, for example, can be shown pictorially more easily than it can be described verbally or in writing. Since graphical representations require few, if any, words, they represent an excellent vehicle for English-language learners to explain concepts. The student work in figure 5.3 is an excellent example of how multiplication can be represented graphically and in words.

The fifth-grade student whose work is illustrated in figure 5.3 is making the transition into English instruction and learning to express thoughts in her second language. By graphically representing the concept as well as practicing the accompanying vocabulary (times, sets, groups, circles), she is able to better demonstrate an understanding of the material. The visual, conceptual, and vocabulary-building elements in this lesson from Math by All Means (Burns 1991) make the unit exceptionally well suited for teaching and assessing English-language learners.
In addition to helping English-language learners, graphics are important tools for all students to represent their mathematical thinking. Their use helps students to visualize ideas and relationships and develop a fuller understanding of underlying concepts. All students should be encouraged to represent responses using graphical depiction, whenever appropriate, but these evaluative procedures are especially important for students with limited language proficiency.

Symbolic Representation

Numbers are tools used to represent mathematical thought symbolically. This symbolic representation is more abstract and therefore more difficult to learn than such concrete representations as manipulatives or drawings. The *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) states that symbolic representations are best learned after concepts have been developed.

Many countries in the world use the Arabic system of numeration, making symbols nearly universal in mathematics. Symbol universality across languages, however, can also encourage teachers to move too quickly to the symbolic expressions before the conceptual foundation has been built. Students’ ability to manipulate symbols without the proper conceptual foundation limits their progress into higher mathematics, since conceptual understanding is the basis for advanced mathematics.

Johnson (1991) describes this relationship between what is known linguistically and what is known conceptually as *logological knowledge*. Once a concept is firmly established through language, the knowledge can become language free as a cognitive structure. This process allows the transfer and application of knowledge in abstract reasoning and noncontextualized situations, which are foundational skills for advanced mathematics. Students who have built a conceptual understanding in their primary language can quickly communicate in the symbolic language of mathematics. However, if students lack a conceptual foundation, one must be developed before they are expected to communicate using the abstract symbolic language. When mathematical learning has been carefully developed over time with a solid base, which can be demonstrated by the students’ ability to describe a concept or process in the language they know best, as Janet did, English-language learners will be ready to master more-complex mathematics with full understanding and long-term retention.
CONCLUSION

It is both a comfort and a challenge to teachers to realize that effective mathematics instruction has much in common with effective second-language teaching. The formula for providing comprehensible input for English-language learners and the strategies used to build mathematical concepts are parallel in their potential for maximizing learning in that they both work on the principle of teaching the unknown from the known. An essential element in making mathematics instruction accessible to English-language learners is careful planning. When teachers analyze the complexity of the concepts being taught and plan a step-by-step development of these concepts along with their corresponding linguistic labels or descriptions, students will benefit through advances in both mathematics and language learning.

The emphasis placed on intellectual development in mathematics by NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) requires that mathematics educators work hand in hand with language educators to ensure a positive interaction of language and cognitive factors as students learn to manipulate complex concepts in increasingly sophisticated linguistic forms. We need not use revolutionary new strategies or invent a new curriculum. We have only to reinforce effective practices by raising teachers' awareness of the concept-language connection.

REFERENCES


