Semester 2 Unit 1 – 1-2 WKST Name: KEY

No Calculator for #1 and #2. A Graphing Calculator is required for #3 and #4.

Find the volume of the figure created by revolving each region bounded by the curves about the given line.

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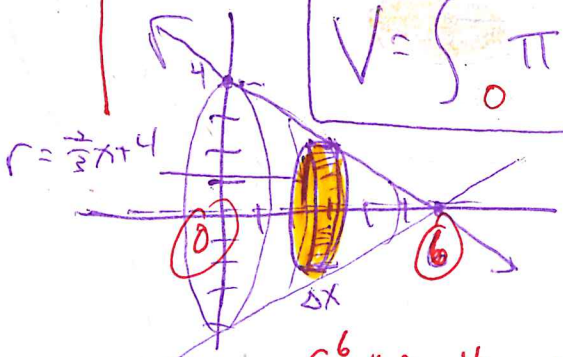
1. $y = -\frac{2}{3}x + 4$, the x-axis, and the y-axis about:

a) the x-axis



$$V = \int_0^6 \pi\left(-\tfrac{2}{3}x+4\right)^2 dx$$

$r = -\frac{2}{3}x+4$

$V = \pi\int_0^6 \frac{4}{9}x^2 - \frac{16}{3}x + 16\,dx$

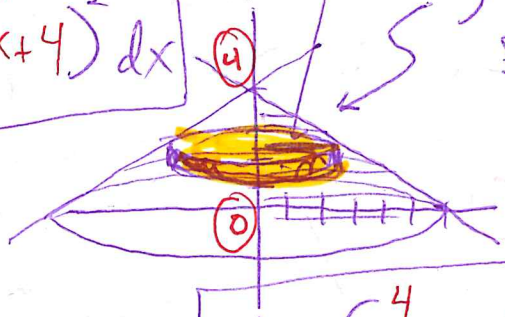
$= \pi\cdot\frac{4}{27}x^3 - \frac{8}{3}x^2 + 16x\Big|_0^6$

$= \pi\left[\frac{4}{27}\cdot 6^3 - \frac{8}{3}\cdot 6^2 + 16\cdot 6 - 0\right]$

$= \pi\,[32 - 96 + 96]$

$= \boxed{32\pi}$

b) the y-axis



$r = x$

$y = -\frac{2}{3}x + 4$

solve for x

$3y = -2x + 12$

$2x = -3y + 12$

$x = -\frac{3}{2}y + 6$ ← r

$$V = \int_0^4 \pi\left(-\tfrac{3}{2}y+6\right)^2 dy$$

FOIL or u-subst

Let $u = -\frac{3}{2}y + 6$

$du = -\frac{3}{2}dy$

$-\frac{2}{3}du = dy$

$\frac{2\pi}{3}\int_6^0 u^2\,du$

$-\frac{2\pi}{3}\cdot\frac{1}{3}u^3\Big|_6^0$

$-\frac{2\pi}{3}\left[0 - \frac{1}{3}\cdot 6^3\right]$

$-\frac{2\pi}{3}[72] \rightarrow \boxed{48\pi}$

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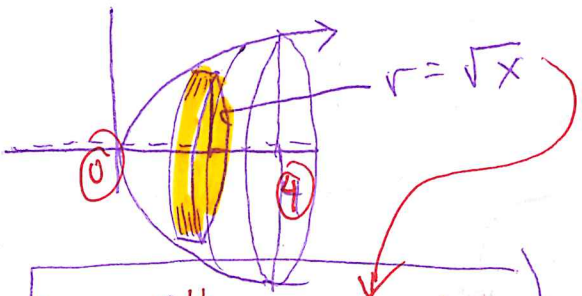
2. $y = \sqrt{x}$, $x = 4$, and the x-axis about:

a) the x-axis



$$V = \int_0^4 \pi\left(\sqrt{x}\right)^2 dx$$

$V = \pi\int_0^4 x\,dx$

$\pi\,\frac{1}{2}x^2\Big|_0^4$

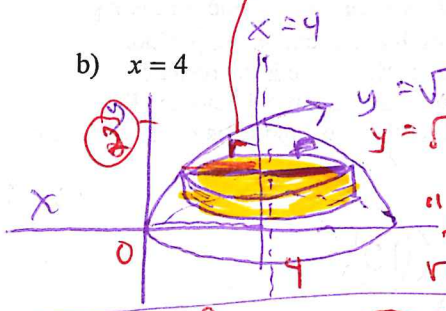
$\pi\left[\frac{1}{2}\cdot 4^2 - 0\right]$

$\boxed{8\pi}$

SOEZ!

No Can u-Subst

b) $x = 4$



solve for x

$x = y^2$

$$V = \int_0^2 \pi\left(\underbrace{4}_{Rt} - \underbrace{y^2}_{left}\right)^2 dy$$

$V = \pi\int_0^2 16 - 8y^2 + y^4\,dy$

$\pi\cdot 16y - \frac{8}{3}y^3 + \frac{1}{5}y^5\Big|_0^2$

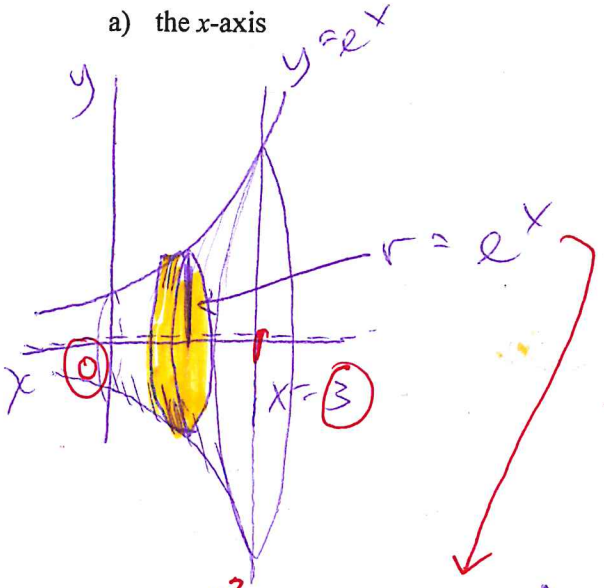
$\pi\left[16\cdot 2 - \frac{8}{3}\cdot 2^3 + \frac{1}{5}\cdot 2^5 - 0\right]$

$\pi\left[32 - \frac{64}{3} + \frac{32}{5}\right]$

$\boxed{\frac{256\pi}{15}}$

3. $y = e^x$, $x = 3$, the x-axis, and the y-axis about:

a) the x-axis



$$V = \int_0^3 \pi (e^x)^2 \, dx$$

b) $x = 3$

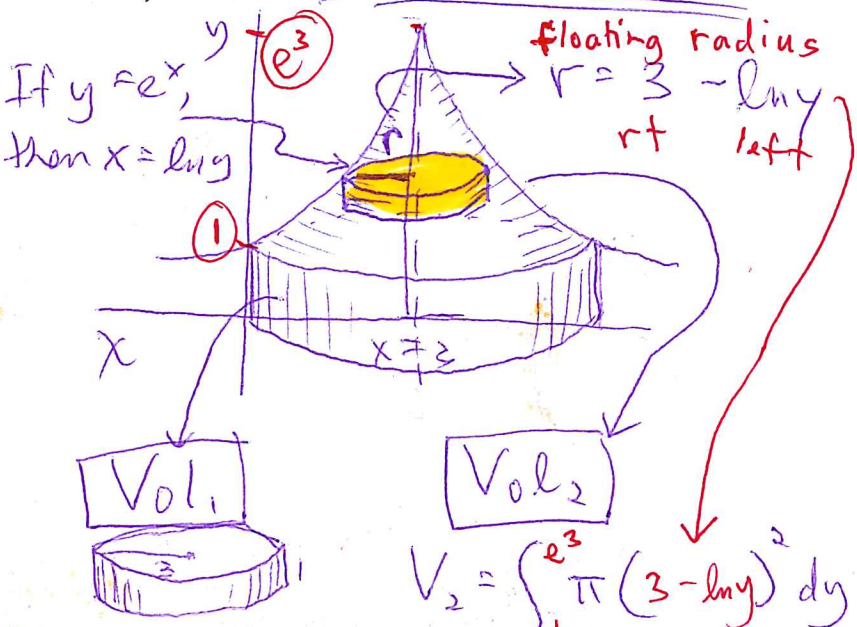
Two Volumes!

floating radius

$r = 3 - \ln y$

rt left

If $y = e^x$, then $x = \ln y$



$$V_2 = \int_1^{e^3} \pi (3 - \ln y)^2 \, dy$$

$V_2 = 72.79408$

$V_1 = \pi 3^2 \cdot 1$

$V_1 = 9\pi$

$\therefore Vol = V_1 + V_2$

$= 101.068$

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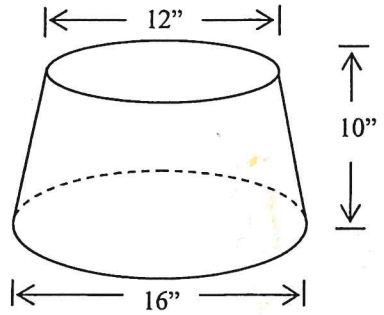
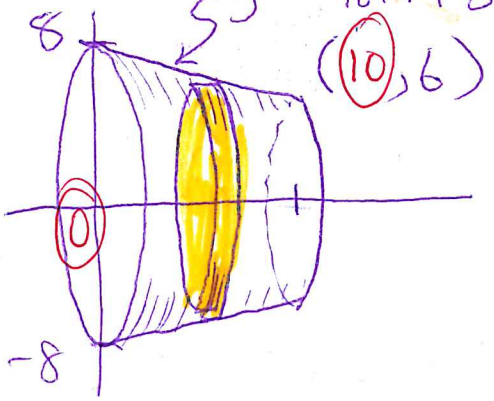
6. Find the volume of the frustum of a cone on the right.

Hint: The figure is a result of a rotation. So position it on the xy-plane in a way so the figure is the result of a line that has been rotated around the x-axis. Find the the equation of that line then apply the definite integral.





$$V = \int_0^{10} \pi \left(-\frac{1}{5}x + 8\right)^2 dx$$

ANSWERS:	
1a) 32π	2a) 632.134
b) 48π	b) 101.068
2a) 8π	6) $1480\pi/3$
b) $256\pi/15$	