Final Exam

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function \( f \) is assumed to be the set of all real numbers \( x \) for which \( f(x) \) is a real number.

(3) The inverse of a trigonometric function \( f \) may be indicated using the inverse function notation \( f^{-1} \) or with the prefix "arc" (e.g., \( \sin^{-1}x = \arcsin x \)).
76. The graph of the function $f$ is shown in the figure above. For which of the following values of $x$ is $f'(x)$ positive and increasing?

(A) $a$     (B) $b$     (C) $c$     (D) $d$     (E) $e$
77. Let \( f \) be a function that is continuous on the closed interval \([2, 4]\) with \( f(2) = 10 \) and \( f(4) = 20 \). Which of the following is guaranteed by the Intermediate Value Theorem?

(A) \( f(x) = 13 \) has at least one solution in the open interval \((2, 4)\).

(B) \( f(3) = 15 \)

(C) \( f \) attains a maximum on the open interval \((2, 4)\).

(D) \( f'(x) = 5 \) has at least one solution in the open interval \((2, 4)\).

(E) \( f'(x) > 0 \) for all \( x \) in the open interval \((2, 4)\).

78. The graph of \( y = e^{\tan x} - 2 \) crosses the \( x \)-axis at one point in the interval \([0, 1]\). What is the slope of the graph at this point?

(A) 0.606  (B) 2  (C) 2.242  (D) 2.961  (E) 3.747
79. A particle moves along the x-axis. The velocity of the particle at time \( t \) is given by \( v(t) \), and the acceleration of the particle at time \( t \) is given by \( a(t) \). Which of the following gives the average velocity of the particle from time \( t = 0 \) to time \( t = 8 \) ?

(A) \( \frac{a(8) - a(0)}{8} \)

(B) \( \frac{1}{8} \int_0^8 v(t) \, dt \)

(C) \( \frac{1}{8} \int_0^8 |v(t)| \, dt \)

(D) \( \frac{1}{2} \int_0^8 v(t) \, dt \)

(E) \( \frac{v(0) + v(8)}{2} \)
80. The graph of $f'$, the derivative of the function $f$, is shown above. Which of the following statements must be true?

I. $f$ has a relative minimum at $x = -3$.

II. The graph of $f$ has a point of inflection at $x = -2$.

III. The graph of $f$ is concave down for $0 < x < 4$.

(A) I only  (B) II only  (C) III only  (D) I and II only  (E) I and III only
81. Water is pumped into a tank at a rate of \( r(t) = 30(1 - e^{-0.16t}) \) gallons per minute, where \( t \) is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A) 380 gallons
(B) 420 gallons
(C) 829 gallons
(D) 1220 gallons
(E) 1376 gallons

82. If \( f'(x) = \sqrt{x^4 + 1 + x^3} - 3x \) then \( f \) has a local maximum at \( x = \)

(A) -2.314 \quad (B) -1.332 \quad (C) 0.350 \quad (D) 0.829 \quad (E) 1.234
83. The graph above gives the velocity, \( v \), in ft/sec, of a car for \( 0 \leq t \leq 8 \), where \( t \) is the time in seconds. Of the following, which is the best estimate of the distance traveled by the car from \( t = 0 \) until the car comes to a complete stop?

(A) 21 ft  (B) 26 ft  (C) 180 ft  (D) 210 ft  (E) 260 ft

84. For \(-1.5 < x < 1.5\), let \( f \) be a function with first derivative given by \( f'(x) = e^{(x^4 - 2x^2 + 1)} - 2 \). Which of the following are all intervals on which the graph of \( f \) is concave down?

(A) \((-0.418, 0.418)\) only
(B) \((-1, 1)\)
(C) \((-1.354, -0.409)\) and \((0.409, 1.354)\)
(D) \((-1.5, -1)\) and \((0, 1)\)
(E) \((-1.5, -1.354)\), \((-0.409, 0)\), and \((1.354, 1.5)\)
85. The graph of \( f' \), the derivative of \( f \), is shown in the figure above. The function \( f \) has a local maximum at \( x = \)

(A) \(-3\)    (B) \(-1\)    (C) \(1\)    (D) \(3\)    (E) \(4\)
86. If $f'(x) > 0$ for all real numbers $x$ and $\int_4^7 f(t) dt = 0$, which of the following could be a table of values for the function $f$?

(A) 
<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
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<tbody>
<tr>
<td>4</td>
<td>-4</td>
</tr>
<tr>
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<tr>
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<td>0</td>
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(B) 
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<tr>
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(C) 
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(D) 
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(E) 
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<td>5</td>
<td>4</td>
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<tr>
<td>7</td>
<td>6</td>
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</tbody>
</table>
87. The graph of \( f'' \), the second derivative of \( f \), is shown above for \(-2 \leq x \leq 4\). What are all intervals on which the graph of the function \( f \) is concave down?

(A) \(-1 < x < 1\)
(B) \(0 < x < 2\)
(C) \(1 < x < 3\) only
(D) \(-2 < x < -1\) only
(E) \(-2 < x < -1\) and \(1 < x < 3\)
88. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person’s shadow is lengthening?

(A) 1.5 ft/sec  
(B) 2.667 ft/sec  
(C) 3.75 ft/sec  
(D) 6 ft/sec  
(E) 10 ft/sec

89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \frac{t + 3}{\sqrt{t^3 + 1}}$. If the particle’s velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713  
(B) 1.134  
(C) 6.134  
(D) 6.710  
(E) 11.710
90. Let \( f \) be a function such that \( \int_{6}^{12} f(2x) \, dx = 10 \). Which of the following must be true?

(A) \( \int_{12}^{24} f(t) \, dt = 5 \)

(B) \( \int_{12}^{24} f(t) \, dt = 20 \)

(C) \( \int_{6}^{12} f(t) \, dt = 5 \)

(D) \( \int_{6}^{12} f(t) \, dt = 20 \)

(E) \( \int_{3}^{6} f(t) \, dt = 5 \)

91. Let \( f \) be a polynomial function with values of \( f''(x) \) at selected values of \( x \) given in the table above. Which of the following must be true for \(-2 < x < 6\)?

(A) The graph of \( f \) is concave up.

(B) The graph of \( f \) has at least two points of inflection.

(C) \( f \) is increasing.

(D) \( f \) has no critical points.

(E) \( f \) has at least two relative extrema.

There's no 92.