$\boxed{\text{FTC}}$ Obj:

to find the total change in the antiderivative function.

FTC $\int_a^b f(x)\,dx = \underbrace{F(b) - F(a)}_{\text{Total } \Delta \text{ in antideriv.}}$

Ex: If $f'(x) = x + 1$ and $f(1) = 2$, find $f(2)$.

<u>Method 1</u> Find $f(x)$ & C. (Particular Soln)

$$f(x) = \tfrac{1}{2}x^2 + x + C$$

$$f(1) = \tfrac{1}{2} + 1 + C = 2$$

$$C = \tfrac{1}{2}$$

$$f(x) = \tfrac{1}{2}x^2 + x + \tfrac{1}{2}$$

$$\therefore f(2) = \tfrac{1}{2}(2)^2 + 2 + \tfrac{1}{2} = \boxed{4.5}$$

Method 2 Using the FTC.

$$\boxed{\int_1^2 \underbrace{x+1}_{f'}\,dx} = f(2) - f(1)$$

$\frac{1}{2}x^2 + x \Big|_1^2$

$\left(\frac{1}{2}\cdot 2^2 + 2\right) - \left(\frac{1}{2} + 1\right)$

$= 2.5$

$2.5 = f(2) - 2$

$4.5 = f(2)$

Ex: If $v(t) = t^2 e^t$ & $s(0) = 5$, then find $s(3)$.

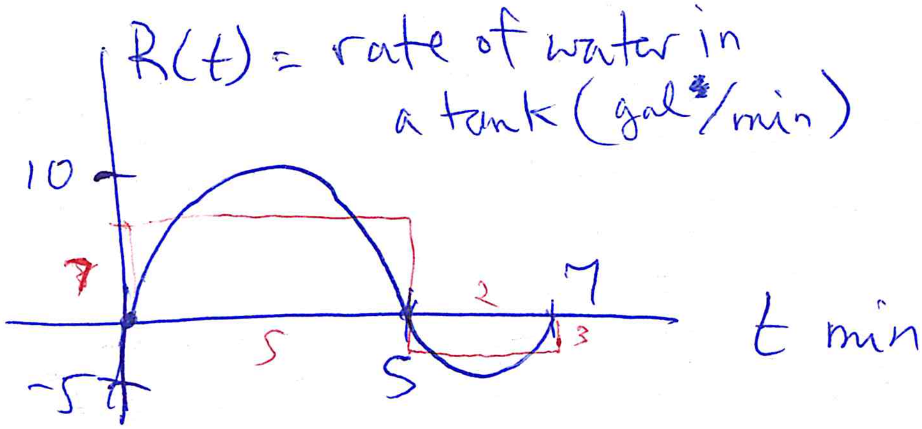
No can find $s(t)$! Thusly, Use FTC!

$$\underbrace{\int_0^3 t^2 e^t\,dt} = s(3) - s(0)$$

$98.428 = s(3) - 5$

$\therefore s(3) = \boxed{103.428}$

Ex :



After 7 min, there was 100 gal in tank, How much water at $t = 0$?

☆ the antiderivative of a Rate is the total change in the y unit of the rate

FTC $$\underbrace{\int_0^7 R(t)\,dt}_{\text{Area of } R(t)} = G(7) - G(0)$$

Use rectangles to estimate area → $5 \cdot 7 - 2 \cdot 3 = 100 - G(0)$

$$29 = 100 - G(0)$$

$$-71 = -G(0)$$

$$\boxed{71 \text{ gal}} = G(0)$$