

## AP Questions Involving The Fundamental Theorem of Calculus

- Total Change in the Antiderivative and Initial Value Problems

Use the Math 9 function on your graphing whenever possible.

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1. Suppose  $R(t) = 3t - 18$  is rate, measured in gallons per minute, at which water is leaking out of a container.

a) Calculate  $\int_0^6 R(t) dt$ .

b) What are the units of  $\int_0^6 R(t) dt$ ?

c) What is the practical meaning of  $\int_0^6 R(t) dt$ ?

d) Suppose there was 100 gallons in the tank initially.

How many gallons are in the tank after 6 minutes?

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2. Suppose  $P(t) = 0.1575(1.032)^t$  is rate, measured in millions per year, at which the population of a country is increasing.

a) Calculate  $\int_0^{10} P(t) dt$ .

b) What are the units of  $\int_0^{10} P(t) dt$ ?

c) What is the practical meaning of  $\int_0^{10} P(t) dt$ ?

d) Suppose the population is currently 5 million.

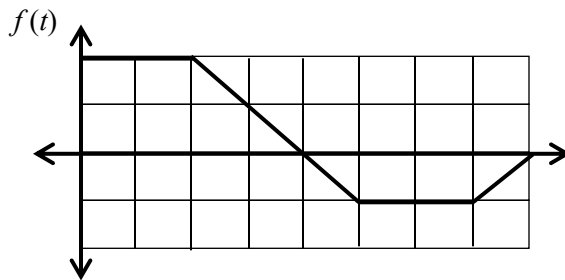
What is the population in 10 years?

$t$	0	2	3	6	8
$r(t)$	12	14	10	12	9

3. For  $0 \leq t \leq 8$  hours, selected values of  $r(t)$ , which is the rate at which a worker produces widgets per hour, is modeled by the table above. Use the trapezoid rule to estimate the amount of widgets that were produced during the 8-hour period?

- (A) 82                      (B) 90                      (C) 92                      (D) 100                      (E) 102

4. Consider the function,  $f(t)$ , below over the interval  $0 \leq t \leq 8$ . Suppose  $f(t) = F'(t)$  and that  $F(0) = 2$ . Find  $F(8)$ .



- (A) -3                      (B) -1                      (C) 2                      (D) 4                      (E) 5

5. Suppose  $f'(x) = x^2$  and  $f(1) = 8$ . Find  $f(3)$ .

- (A)  $\frac{50}{3}$                       (B)  $\frac{25}{3}$                       (C)  $\frac{100}{9}$                       (D)  $\frac{25}{6}$                       (E)  $\frac{50}{9}$

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6. A particle moves along the  $y$ -axis so that at any time  $t > 0$ , its velocity is given by  $v(t) = -3.4 \sin(0.2t - \pi)$ . If the position of the particle is at  $y = 6$  at time  $t = 3$ , then find the initial position of the particle.

- (A) 3.031                      (B) 3.234                      (C) 3.509                      (D) 3.750                      (E) 3.983

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7. A bug crawls in a straight path along the floor so that its acceleration is given by  $a(t) = t^{-0.5}(1 + e^{-t})$  for  $t > 0$ . If the velocity of the particle is 4 cm/sec at time  $t = 1$ , then the velocity of the particle at time  $t = 5$  is

- (A) 4.434 cm/sec              (B) 4.920 cm/sec              (C) 5.303 cm/sec              (D) 5.824 cm/sec              (E) 6.748 cm/sec

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8. A cold ham, having the temperature of  $42^\circ\text{F}$  (Fahrenheit), is heated in an oven set to  $425^\circ\text{F}$ . The temperature of the ham is changing at a rate of  $134.05 e^{-0.35t}$  degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the ham at 15 minutes?

- (A)  $412^\circ$                       (B)  $416^\circ$                       (C)  $419^\circ$                       (D)  $423^\circ$                       (E)  $425^\circ$
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9. A baked ham, heated to a temperature of  $425^{\circ}\text{F}$ , is placed in a refrigerator at the temperature of  $42^{\circ}\text{F}$ . The temperature of the ham is changing at a rate of  $-76.6e^{-0.2t}$  degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the ham at 15 minutes?

- (A)  $93.833^{\circ}$             (B)  $76.352^{\circ}$             (C)  $61.068^{\circ}$             (D)  $55.784^{\circ}$             (E)  $49.015^{\circ}$

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10. A car going  $80\text{ft/sec}$  (about  $55\text{mph}$ ) brakes to a stop in five seconds. Assuming the deceleration is constant, find the car's stopping distance. Stopping distance is the distance traveled from the time the car applies the brakes to when it stops. (Hint: Sketch the graph of the velocity.)

- (A) 90 ft            (B) 120 ft            (C) 150 ft            (D) 200 ft            (E) 220 ft

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11. Suppose a car going at  $30\text{ft/sec}$  decelerates at a constant rate of  $5\text{ft/sec}^2$ . Find the car's stopping distance.

- (A) 90 ft            (B) 120 ft            (C) 150 ft            (D) 200 ft            (E) 220 ft
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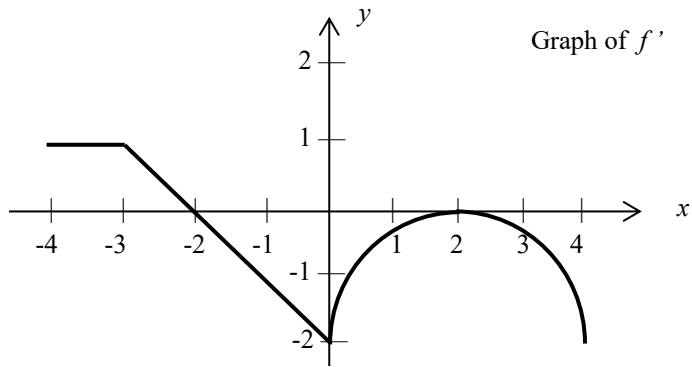
12. A car, initially moving at 75 ft per second, slows at a constant rate and has a stopping distance of 250 feet. What is its “acceleration” of the car during its stopping time?

- (A) -11.25            (B) -12.75            (C) -13.25            (D) -14.5            (E) -15

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13. The rate of change of the population of a school is given by  $r(t) = -t^3 + 4t^2 - 6$  for  $0 \leq t \leq 5$ . Which of the following expressions gives the total change in population during the time the number of students is increasing?

- (A)  $\int_{1.572}^{3.514} r'(t) dt$     (B)  $\int_0^{2.667} r'(t) dt$     (C)  $\int_0^{2.667} r(t) dt$     (D)  $\int_{1.572}^{3.514} r(t) dt$     (E)  $\int_0^5 r(t) dt$
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14. Let  $f$  be a function defined on the closed interval  $-4 \leq x \leq 4$  with  $f(0) = 4$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle, as shown above.
- On what intervals, if any is  $f$  increasing? Justify your answer.
  - Find the  $x$ -coordinate of each point of inflection of the graph of  $f$  on the open interval  $-4 < x < 4$ . Justify your answer.
  - Find the equation for the line tangent to the graph of  $f$  at the point  $(0, 4)$ .
  - Find  $f(-4)$  and  $f(4)$ . Show the work that leads to your answers.

**ANSWERS:**

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|----------------------------------|-------------------------------|------|-------|---|
| 1a) - 54                         | 2a) 1.851                     | 4) E | 9) C  | 14a) $-4 < x < -2$ , since $f' > 0$ on $-4 < x < -2$        |
| b) gallons                       | b) millions                   | 5) A | 10) D | b) At $x = 0$ and $x = 2$ , since there are changes in $f'$ |
| c) total gallons that leaked out | c) total change in population | 6) A | 11) A | from decrease to increase at $x = 0$ and from               |
| d) 46 gallons                    | d) 6.851 million              | 7) E | 12) A | increase to decrease at $x = 2$ , which means $f''$         |
|                                  | 3) C                          | 8) D | 13) D | changes signs at the values $x = 0$ and $x = 2$ .           |
|                                  |                               |      |       | c) $y = -2x + 4$  |
|                                  |                               |      |       | d) $f(-4) = 4.5, f(4) = 2\pi - 4$                           |