5. Consider the differential equation \( \frac{dy}{dx} = \frac{y-1}{x^2} \), where \( x \neq 0 \).

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

(b) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(2) = 0 \).

(c) For the particular solution \( y = f(x) \) described in part (b), find \( \lim_{x \to \infty} f(x) \).
5. Consider the differential equation \( \frac{dy}{dx} = x^4 (y - 2) \).

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.  
(Note: Use the axes provided in the test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the \( xy \)-plane. Describe all points in the \( xy \)-plane for which the slopes are negative.

(c) Find the particular solution \( y = f(x) \) to the given differential equation with the initial condition \( f(0) = 0 \).
5. Consider the differential equation \( \frac{dy}{dx} = \frac{1 + y}{x} \), where \( x \neq 0 \).

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)

(b) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(-1) = 1 \) and state its domain.
5. Consider the differential equation \( \frac{dy}{dx} = (y - 1)^2 \cos(\pi x) \).

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

(b) There is a horizontal line with equation \( y = c \) that satisfies this differential equation. Find the value of \( c \).

(c) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(1) = 0 \).
Question 5

Consider the differential equation \( \frac{dy}{dx} = \frac{y-1}{x^2} \), where \( x \neq 0 \).

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
   (Note: Use the axes provided in the exam booklet.)

(b) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(2) = 0 \).

(c) For the particular solution \( y = f(x) \) described in part (b), find \( \lim_{x \to \infty} f(x) \).

\[ \frac{1}{y-1} \, dy = \frac{1}{x^2} \, dx \]
\[ \ln|y-1| = -\frac{1}{x} + C \]
\[ |y-1| = e^{-\frac{1}{x} + C} \]
\[ |y-1| = e^C e^{-\frac{1}{x}} \]
\[ y-1 = ke^{-\frac{1}{x}}, \text{ where } k = \pm e^C \]
\[ -1 = ke^{-\frac{1}{2}} \]
\[ k = -e^{\frac{1}{2}} \]
\[ f(x) = 1 - e^{\left(\frac{1}{x} - \frac{1}{2}\right)}, \ x > 0 \]

\[ \lim_{x \to \infty} 1 - e^{\left(\frac{1}{x} - \frac{1}{2}\right)} = 1 - \sqrt{e} \]
Question 5

Consider the differential equation \( \frac{dy}{dx} = x^4(y - 2) \).

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (Note: Use the axes provided in the test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the \(xy\)-plane. Describe all points in the \(xy\)-plane for which the slopes are negative.

(c) Find the particular solution \( y = f(x) \) to the given differential equation with the initial condition \( f(0) = 0 \).

(a)

\[
\begin{array}{c}
\text{1: zero slope at each point } (x, y) \\
\text{where } x = 0 \text{ or } y = 2 \\
\text{2: positive slope at each point } (x, y) \\
\text{where } x \neq 0 \text{ and } y > 2 \\
\text{1: negative slope at each point } (x, y) \\
\text{where } x \neq 0 \text{ and } y < 2 \\
\end{array}
\]

(b) Slopes are negative at points \((x, y)\) where \(x \neq 0\) and \(y < 2\).

(c) \[
\frac{1}{y - 2} dy = x^4 dx \\
\ln|y - 2| = \frac{1}{5} x^5 + C \\
|y - 2| = e^C e^{\frac{1}{5} x^5} \\
y - 2 = K e^{\frac{1}{5} x^5}, \quad K = \pm e^C \\
-2 = K e^0 = K \\
y = 2 - 2 e^{\frac{1}{5} x^5}
\]
Question 5

Consider the differential equation \( \frac{dy}{dx} = \frac{1 + y}{x} \), where \( x \neq 0 \).

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated. (Note: Use the axes provided in the pink exam booklet.)

(b) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(-1) = 1 \) and state its domain.

\[
\begin{align*}
\frac{1}{1 + y} \frac{dy}{dx} &= \frac{1}{x} \\
\ln |1 + y| &= \ln |x| + K \\
|1 + y| &= e^{\ln |x| + K} \\
1 + y &= C|x| \\
2 &= C \\
1 + y &= 2|x| \\
y &= 2|x| - 1 \text{ and } x < 0 \\
or \\
y &= -2x - 1 \text{ and } x < 0
\end{align*}
\]
Question 5

Consider the differential equation \( \frac{dy}{dx} = (y - 1)^2 \cos(\pi x) \).

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.  
   (Note: Use the axes provided in the exam booklet.)

(b) There is a horizontal line with equation \( y = c \) that satisfies this differential equation.  Find the value of \( c \).

(c) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(1) = 0 \).

---

(a) The line \( y = 1 \) satisfies the differential equation, so \( c = 1 \).

(c) \[
\frac{1}{(y - 1)^2} \frac{dy}{dx} = \cos(\pi x) \frac{dx}{dx}
\]
   \[-(y - 1)^{-1} = \frac{1}{\pi} \sin(\pi x) + C
\]
   \[
\frac{1}{1 - y} = \frac{1}{\pi} \sin(\pi x) + C
\]
   \[
1 = \frac{1}{\pi} \sin(\pi) + C = C
\]
   \[
\frac{1}{1 - y} = \frac{1}{\pi} \sin(\pi x) + 1
\]
   \[
\frac{\pi}{1 - y} = \sin(\pi x) + \pi
\]
   \[
y = 1 - \frac{\pi}{\sin(\pi x) + \pi} \text{ for } -\infty < x < \infty
\]

2 : \{ 1 : zero slopes  
2 : all other slopes

1 : \[ c = 1 \]

1 : separates variables  
2 : antiderivatives  
6 : \{ 1 : constant of integration  
1 : uses initial condition  
1 : answer

Note: max 3/6 [1-2-0-0-0] if no constant of integration  
Note: 0/6 if no separation of variables