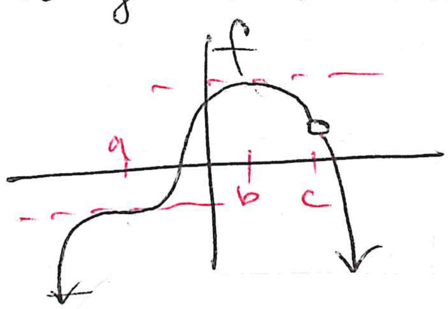
3.3 Day 1 & Day 2

Obj: to find critical numbers, intervals of incr/decr, & to use the First Derivative Test to find Relative Extrema.

Critical numbers

horz. tangent $\Rightarrow f'(x) = 0$ or $f'(x)$ undefined



Eg: Find the critical numbers of

$$\boxed{f(x) = x^3 - \frac{3}{2}x^2}$$

★ To find critical numbers, find $f'(x)$ & set $= 0$.

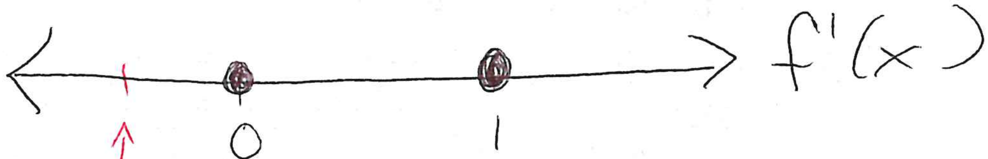
$$f'(x) = 3x^2 - 3x = 0$$

$$3x(x-1) = 0$$

$\boxed{x = 0}$ $\boxed{x = 1}$ ← Crit. #s

Eg: Find where $f(x) = x^3 - \frac{3}{2}x^2$ increases or decreases

☆ To find Intervals of Increase and decrease, Plot critical numbers on a $f'(x)$ number line, then test a number in each interval to see if the value is positive or negative.



choose a number & plug into $f'(x) = 3x(x-1)$ ← Use the factored form.

$x = -1$.

$f'(-1) = 3(-1)(-1-1)$

$= -3(-2)$

$f'(-1) = 6$

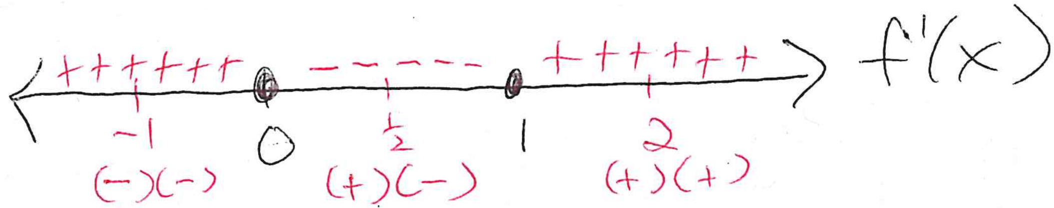
OR $f'(-1) = 3(-1)(-1-1)$

$(-)(-)$

$(+)$

Note: ← You don't need to calculate the actual value. You just need to see if $f'(1)$ is pos. or neg.

Doing the same for the other intervals,



∴ $\boxed{\begin{array}{l} f(x) \text{ incr on } (-\infty, 0) \cup (1, +\infty) \\ f(x) \text{ decr on } (0, 1) \end{array}}$

Eg: Use the First Derivative Test to determine the relative extrema of $f(x) = x^3 - \frac{3}{2}x^2$.

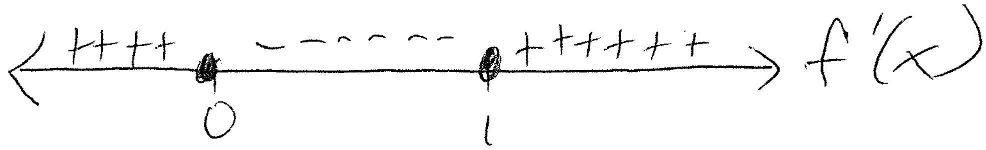
First Derivative test

$x = c$ is $\boxed{\text{rel max}}$ if $f'(x)$ changes from pos to neg ∴ $\boxed{x = 0 \text{ rel max}}$

$x = c$ is $\boxed{\text{rel min}}$ if $f'(x)$ changes from neg to pos ∴ $\boxed{x = 1 \text{ rel min}}$

$x = c$ is $\boxed{\text{neither}}$ if $f'(x)$ has no sign change.

∴ Looking at the number line



$x=0$ is a relative max since $f'(x)$ changes from pos to neg at $x=0$.

$x=1$ is a relative min since $f'(x)$ changes from neg to pos at $x=1$.