Chapter 3 TEST – AP Calculus

Name: _____ PT

Practice

Part A **No Calculator**

1. If $f(2) = 7$ and $f'(2) = 0$, then which of the following must be true?

   I. $f(x)$ has a local extreme value when $x = 2$
   II. $f(x) = 7$ for all x
   III. $(2, 7)$ is a point of inflection

   (A) None (B) I only (C) II only
   (D) III only (E) I and III only

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2. If f is differentiable for all x, and has a local maximum at $x = 3$, then which of the following must be true?

   I. $f'(3) = 0$
   II. $f''(3) \leq 0$
   III. f is continuous at $x = 3$

   (A) I only (B) II only (C) I and II only
   (D) I and III only (E) I, II, and III

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3. What are all the x-coordinates of the critical points for the graph of $f(x) = (x-4)(x-2)$?

   (A) 2 and 4 (B) 0, 2, and 4 (C) 2, 3, and 4 (D) 0 and 3 (E) 3

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4. If f is continuous of the closed interval $[1, 3]$ with $f'(x) < 0$ on the open interval $(1, 3)$, then

   (A) $f(x)$ does not have a minimum on $[1, 3]$
   (B) $f(x)$ does not have a maximum on $[1, 3]$
   (C) $f(1)$ is the maximum value of $f(x)$ on $[1, 3]$
   (D) $f(1)$ is the minimum value of $f(x)$ on $[1, 3]$
   (E) $f(3)$ is the maximum value of $f(x)$ on $[1, 3]$

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5. If f and f' are continuous for all x, and if f has a local maximum at $x = 4$, then which of the following must be true?

I. $f(4) > f(5)$
II. $f'(4) = 0$
III. $f''(4) < 0$

(A) I only (B) II only (C) I and II only
(D) II and III only (E) I, II, and III

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6. If f, f', and f'' are continuous for all x with $f(5) = 9$, and if $f'(x)$ has a local maximum at $x = 5$, then

(A) $f'(5) = 0$

(B) the graph of f is concave up in an open interval containing $x = 5$

(C) f is increasing in an open interval containing $x = 5$

(D) the point $(5, 9)$ is a point of inflection

(E) $f''(5) < 0$

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7. If $f(x) = x^2 + 2x - 8$, then $f(x)$ has a local minimum at

(A) $x = 2$ only

(B) $x = 2$ and $x = -4$

(C) $x = -4$ only

(D) $x = -1$ only

(E) $x = -1$ and $x = 2$

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8. If $f(x) = xe^{-x}$, then the critical points of f are

(A) $x = -1$ only

(B) $x = 0$ only

(C) $x = 1$ only

(D) $x = 0$ and $x = -1$

(E) $x = 0$ and $x = 1$

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9. The foot of a 20 ft ladder is being pulled away from a wall at the rate of 1.5 ft/sec. At the instant when the foot is 12 ft away from the wall, the angle the ladder makes with the floor is decreasing at the rate (in radians/sec) of:

(A) $\frac{3}{50}$ (B) $\frac{1}{16}$ (C) $\frac{3}{40}$ (D) $\frac{1}{8}$ (E) $\frac{3}{32}$

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10. If f and g are differentiable for all x, $h(x) = f(x) - g(x)$, and $h(x)$ has a local maximum at $x = 3$, then

(A) $f(x)$ has a relative maximum value at $x = 3$

(B) $g(x)$ has a relative minimum value at $x = 3$

(C) $f'(3) > g'(3)$

(D) $f'(3) = g'(3)$

(E) $f'(3) < g'(3)$

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11. If $3p + 2q = 600$, the maximum value of $p \cdot q$ is

(A) 100 (B) 150 (C) 600 (D) 15,000 (E) 60,000

x	$F(x)$	$F'(x)$	$F''(x)$
2	1	3	-4
4	3	0	-2
6	7	5	0
8	13	6	2

12. The table above gives some information about a function F, for which F, F', and F'' are continuous for all x. Both F' and F'' have exactly one zero on the interval [2 , 8]. Which of the following statement(s) must be true?

(A) $F(x)$ is increasing on the interval [2 , 8].

(B) The point (6 , 7) is a point of inflection.

(C) $F(x)$ has a local minimum when $x = 4$.

(D) The line $y = 7$ is a horizontal asymptote of $F(x)$.

(E) The line $x = 5$ could be a vertical asymptote of $F(x)$.

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13. Let $f(x) = \dfrac{\sin x}{e^x}$ for $x > 0$. When the minimum value of $f(x)$ occurs, then

(A) $\sin x = 0$

(B) $\cos x = 0$

(C) $\cos x = \sin x$

(D) $\cos x = -\sin x$

(E) $f(x)$ foes not any extreme values on the interval [0 , ∞).

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14. If $f'(x) = (x-1)^2(x+4)$, then the function $y = f(x)$ has

(A) one local minimum and no local maximum

(B) no local minimum and one local maximum

(C) one local minimum and one local maximum

(D) one local minimum and two local maxima

(E) two local minima and one local maximum

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15. If $f''(x) = (x-1)^2(x-3)\cos x$, then on the interval $(0, \pi)$, how many points of inflection does the graph of $y = f(x)$ have?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

16. If $f(x) = \dfrac{\ln x}{x}$, then

(A) $f(x)$ has a local maximum when $x = 1$

(B) $f(x)$ has a local minimum when $x = 1$

(C) $f(x)$ has a local maximum when $x = e$

(D) $f(x)$ has a local minimum when $x = e$

(E) $f(x)$ has no local extreme values

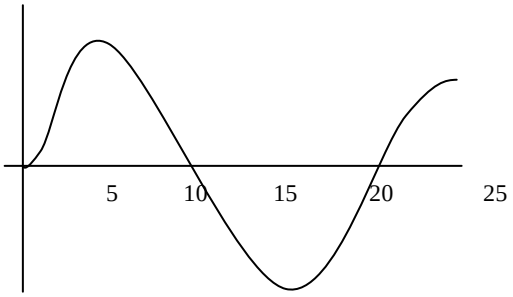
17. The graph of $y = x + \dfrac{1}{x}$ is both increasing and concave down on the interval

(A) $(-\infty, -1)$ (B) $(-1, 0)$ (C) $(0, 1)$ (D) $(1, \infty)$ (E) never

Part A **No Calculator**

18. If $f'(x) = (x-1)^2(x+2)$, then f is increasing and concave down on the interval

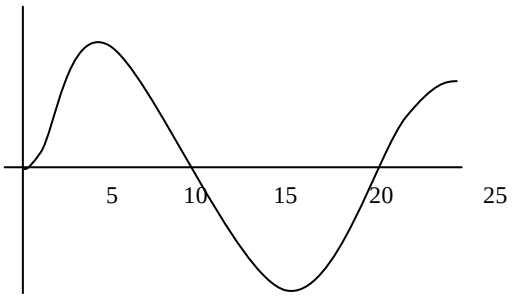
(A) $(-\infty, -2)$ (B) $(-2, -1)$ (C) $(-2, 1)$ (D) $(-1, 1)$ (E) $(1, \infty)$



19. The graph of $y = f'(x)$, the derivative of f, is shown above. The function $y = f(x)$ has a local minimum on the interval $(0, 25)$ when x equals

(A) 10 only (B) 15 only (C) 20 only

(D) 4 and 15 only (E) 10 and 20



20. The graph of $y = f'(x)$, the derivative of f, is shown above. The function $y = f(x)$ is concave down on the interval(s)

(A) $(0, 4)$ (B) $(4, 15)$ (C) $(10, 20)$

(D) $(15, 25)$ (E) $(0, 10)$ and $(20, 25)$

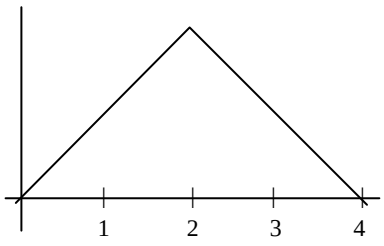
21. If $f(x)$ and $g(x)$ are both increasing differentiable functions defined for all x, then which of the following must be true?

I. $f(x)+g(x)$ is increasing
II. $f(x)\cdot g(x)$ is increasing
III. $f(g(x))$ is increasing

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

22. If $y=f(x)$ is an increasing differentiable function whose graph lies in the first quadrant and is concave down, then $g(x)=\dfrac{1}{f(x)}$ is a function that is

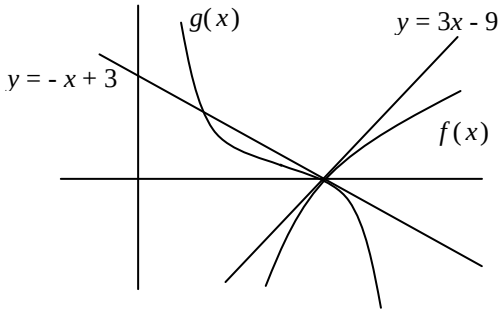
(A) increasing and concave down
(B) decreasing and concave down
(C) increasing and concave up
(D) decreasing and concave up
(E) not enough information to determine



23. The graph of $y=f'(x)$, the derivative of f, is shown above. Which of the following is true?

I. $y=f(x)$ has a point of inflection when $x=2$
II. The maximum value of $f(x)$ occurs when $x=2$
III. The maximum value of $f(x)$ occurs when $x=4$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only



Hint: Use L'hopital's Rule.

If $f(c) = 0$ and $g(c) = 0$,

then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)} = \frac{f'(c)}{g'(c)}$$

24. The functions f and g and their tangent lines at (3 , 0) are shown above.

$$\lim_{x\to 3}\frac{f(x)}{g(x)} =$$

(A) -6 (B) -3 (C) $-\frac{1}{3}$ (D) $\frac{1}{3}$ (E) 3

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25. The rate of change of the volume, V, of water in a tank with respect to time, t, is inversely proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$ (B) $V(t) = k\sqrt{V}$ (C) $\frac{dV}{dt} = k\sqrt{t}$ (D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$ (E) $\frac{dV}{dt} = k\sqrt{V}$

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x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

26. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is increasing on which of the following intervals?

(A) $-2 \le x \le 2$ only (B) $-1 \le x \le 1$ only (C) $x \ge -2$

(D) $x \ge 2$ only (E) $x \le -2$ or $x \ge 2$

PT

Part A **No Calculator**

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27. Let $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f increasing?

(A) $(-\infty, -1]$ only (B) $(-\infty, 0)$ or $[\sqrt[3]{2}, \infty)$ (C) $[-1, 0)$ only

(D) $(-\infty, \sqrt[3]{2}]$ (E) $[\sqrt[3]{2}, \infty)$ only

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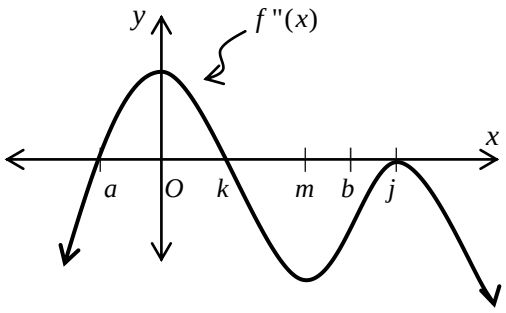

28. The second derivative of the function f is given by $f''(x) = -x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) k and b only (B) 0 and m only (C) a, k and j (D) 0, m, and j (E) a and k only

[Just for fun, what would the answer be if the graph was $f'(x) = -x(x-a)(x-b)^2$?]

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29. A particle moves along the x-axis so that at time $t \geq 0$, its position is given by $x(t) = \frac{4}{3}t^3 - 14t^2 + 49t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

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x	$f(x)$	$f'(x)$
1	2	1/2
2	3	1/3
3	1	-2

30. The table above shows selected values of $f(x)$ and $f'(x)$. If f and f^{-1} (the inverse of f) exist, are continuous and differentiable for $x > 0$, then $\frac{d}{dx}\left(f^{-1}(x)\right)$ at $x = 1$ is

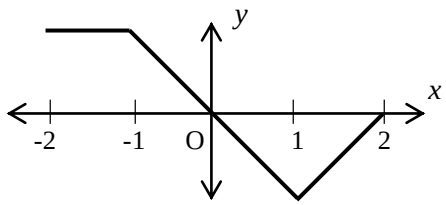
(A) -4 (B) -2 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 2

For #33 to #35, evaluate each limit and then identify any horizontal asymptotes.

31. $\lim_{x\to\infty} \frac{x^3+8x-4}{2x^3+3}$

32. $\lim_{x\to\infty} \frac{x^2-1}{2x^3-8x^2+3}$

33. $\lim_{x\to\infty} \frac{x^5-x-1}{6x^3}$



Graph of f'

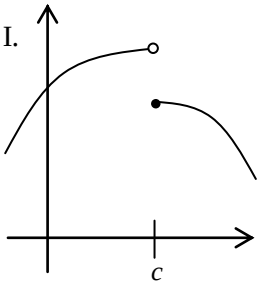
34. The graph of f', the derivative of the function f, is shown above. Which of the following statements is **FALSE** about f ?

(A) f is concave down for $-1 \le x \le 1$
(B) f is decreasing for $0 \le x \le 2$
(C) f is increasing for $-2 \le x \le 0$
(D) f has a maxmum at $x = 0$
(E) f has a point of inflection at $x = -1$ and $x = 1$

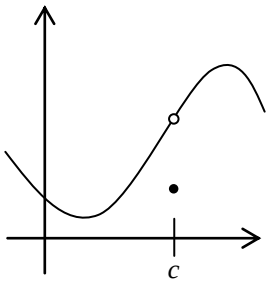
Part A **No Calculator**

35. For which of the following does $\lim_{x \to c} f(x)$ exist?

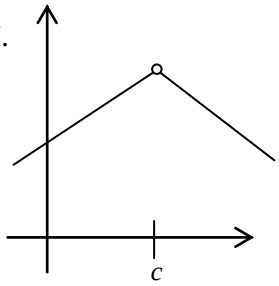
I.



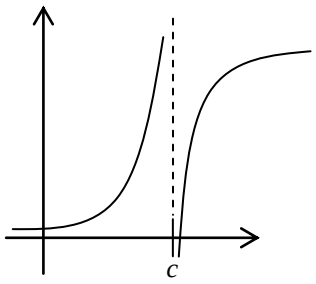
II.



III.



IV.



(A) I only (B) II only (C) IV only (D) II and III only (E) I , II, and III

36. What are all horizontal asymptotes of the graph of $f(x) = \frac{5x^3+10}{6(2^x)-x^3}$ in the *xy* -plane?

(A) $y = \frac{5}{6}$ only (B) $y = -5$ only (C) $y = 0$ only
(D) both $y = 0$ and $y = -10$ (E) both $y = 0$ and $y = -5$

37. Which of the following are not differentiable on the interval $[-3, 3]$?

I. $f(x) = cos2x - e^x$
II. $f(x) = \sqrt[3]{x-2}$
III. $f(x) = |x - 3|$
IV. $f(x) = |x| - 3$

(A) I only (B) II only (C) II and III (D) II and IV (E) III and IV

Part A **No Calculator**

x	0	2	4
$f(x)$	1	4	k

38. The function f is continuous on the closed interval [0 , 4] and has values that are given in the table above. If $f'(x) > 0$ and $f''(x) < 0$, then k could be

(A) 4 (B) 6 (C) 7 (D) 8 (E) 10

39. Evaluate the following limit. $\lim_{h\to 0} \frac{\sqrt{9+h}-\sqrt{9}}{h}$ =

(A) 1 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 3 (E) 6

40. Consider the piece-wise function $f(x) = \begin{cases} cosx & x \le 0 \\ 1 & x > 0 \end{cases}$.

I. $\lim_{x\to 0} f(x) = 1$

II. $f(x)$ is differentiable at $x = 0$

III. $\lim_{x\to 0} f(x) = f(0)$

IV. $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$ exists

Which of the following are true?

(A) none (B) I only (C) I and III (D) II and IV (E) I, II, III, and IV

Part A **No Calculator**

Use the table of values for $f(t)$ to answer #43 to #45.

t	0	1	2	3	4
$f(t)$	0	2	6	12	20

41. Find the best approximation for $f'(1)$

42. Find the best approximation for $f'(2.5)$

43. Find the best approximation for $f'(4)$

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44. Find the instantaneous rate of change at $x = 3$ for $x^3 + x^2y = e$.

(A) $\frac{-2e-27}{27}$

(B) $\frac{-81-2e}{27}$

(C) $\frac{-2e+54}{9}$

(D) $\frac{-2e}{9}$

(E) $\frac{-2e}{3}$

Part A **No Calculator**

45. If $f(x) = x + e^{2x}$, then which of the following is an equation of the line tangent to the graph of f at the point where $x = 0$?

(A) $y = 2x + 1$ (B) $y = 2x$ (C) $y = 3x + 1$
(D) $y = 3x$ (E) $y = 2x + 2$

46. If $f(x) = x^3 + 1$ and $g(x) = f^{-1}(x)$ and $g(9) = 2$, $g'(9)$.

(A) $\frac{1}{2}$ (B) $\frac{1}{9}$ (C) $\frac{1}{12}$ (D) 1 (E) 2

PT

Part A — No Calculator

47. Find $\left.\frac{dy}{d\theta}\right|_{\theta=\frac{\pi}{6}}$ for $y = sin2\theta\ tan\theta$.

(A) 1 (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{4}$ (D) $2\sqrt{3}$ (E) $\sqrt{3}$

Stop! You may use your graphing calculator for the remainder of the test.

Part B — Graphing Calculator Required

76. The position of a particle moving vertically for $t \geq 0$ is given by $x(t) = 4t - 4.9sec^2(3t)$. The velocity of the particle at time $t = 2$ is

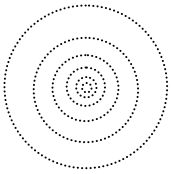
(A) -12.106
(B) -0.725
(C) 12.071
(D) 34.808
(E) 13.280

Part B **Graphing Calculator Required**

77. A pebble thrown into a pond creates circular ripples such that the rate of change of the circumference is 12π cm/sec. How fast is the area of the ripple changing when the radius is 3 cm?

(A) 6π cm²/sec
(B) 2π cm²/sec
(C) 12π cm²/sec
(D) 36π cm²/sec
(E) 6 cm²/sec

Hint: $A = \pi r^2$
$C = 2\pi r$



78. The velocity of a particle moving along the x-axis is $v(t) = t^2 \cos t$. How many relative extrema does the particle experience on the interval $0 \le t \le 5$?

(A) One (B) Two (C) Three (D) Four (E) Five

79. A The velocity of a particle moving along the x-axis is $v(t) = t^2 \cos t$. When will the particle be the farthest to the left on the interval $0 \le t \le 5$?

(A) $t = 0$ (B) $t = 1.570$ (C) $t = 3.644$ (D) $t = 4.712$ (E) $t = 5$

Part B **Graphing Calculator Required**

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80. Let $f(x) = 3x^4$ and $g(x) = e^{3x-4}$. At what value of x does f and g have the same rate of change?

(A) 0.127 (B) 0.204 (C) 0.455 (D) 0.649

(E) There are no such values.

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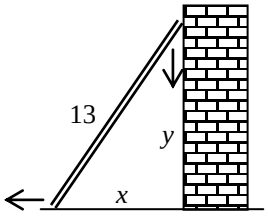
81. For the equation $y = 2x^4 - 4x^2 - 5$ on $x > 0$, there is a value $x = c$ such that $\left.\frac{dy}{dx}\right|_{x=c} = 1$. The equation at the tangent line at $x = c$ is

(A) $y = x - 4.939$
(B) $y = x - 5.062$
(C) $y = x - 6.501$
(D) $y = x - 6.852$
(E) $y = x - 7.598$

Part B **Graphing Calculator Required**

82. A ladder 13 feet long is leaning against a wall. If the foot of the ladder is pulled away from the wall at the rate of 0.5 feet per second, how fast will the top of the ladder be dropping at the instant when the base is 5 feet from the wall?



(A) $-\frac{1}{12}$ ft/sec

(B) $-\frac{1}{8}$ ft/sec

(C) $-\frac{1}{6}$ ft/sec

(D) $-\frac{5}{24}$ ft/sec

(E) $-\frac{1}{4}$ ft/sec

83. Use the same situation in problem #82. Consider the area of the triangle created by the ladder and the wall. How fast is this area changing at the instant when the base is 5 feet from the wall?

(A) $\frac{49}{4}$ ft²/sec (B) $\frac{27}{6}$ ft²/sec (C) $\frac{152}{7}$ ft²/sec (D) $\frac{193}{64}$ ft²/sec (E) $\frac{119}{48}$ ft²/sec

Part B **Graphing Calculator Required**

84. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$

(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$

(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$

(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$

(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$

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85. The function $f(x)$ is continuous for the closed interval [a , b] and differentiable for the open interval (a , b). If $f(a) = f(b)$ and $f'(x)$ changes sign only once on [a , b], then which of the following could be false?

(A) $\lim\limits_{x \to c} f(x) = f(c)$ for all values c such that $a < c < b$.

(B) f has a minimum or maximum value on $a \le x \le b$.

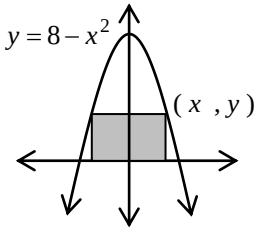
(C) f has a point of inflection on $a \le x \le b$.

(D) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(E) $f'(c) = 0$ for some c such that $a < c < b$.

Part B **Graphing Calculator Required**

86. Find the <u>*x*-coordinate</u> that produces the largest possible area for a rectangle with its base on the *x*-axis and upper vertices on the curve $y = 8 - x^2$. Use Calculus concepts to solve.



Hint: The base of the rectangle is not x. The base is $2x$.

(A) $\frac{\sqrt{6}}{3}$ units

(B) $\frac{2\sqrt{6}}{3}$ units

(C) $\sqrt{6}$ units

(D) $\frac{4\sqrt{6}}{3}$ units

(E) $\frac{5\sqrt{6}}{3}$ units

87. A rectangle of perimeter 18 inches is rotated about one of its sides to generate a right circular cylinder. What is the area in square inches of the rectangle that generates the cylinder with the largest volume?
Use Calculus concepts to solve.

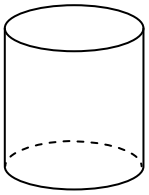
(A) 10 (B) 12 (C) 15 (D) 16 (E) 18

Part B **Graphing Calculator Required**

88. You are to make a one-pint oil can shaped like a right circular cylinder. Find the radius that will use the least material. Use Calculus concepts to solve.

Hints: 1 pint = 28.875 cubic inches

$V = \pi r^2 h$ $SA = 2\pi r^2 + 2\pi r h$



(A) 1.663 inches
(B) 1.798 inches
(C) 1.937 inches
(D) 2.049 inches
(E) 2.325 inches

Part B **Graphing Calculator Required**

For the following functions, i) identify the critical points, ii) classify the critical points as relative maxima or minima, and iii) identify the points of inflection on $f(x)$.

89. $f'(x) = 3x^4 - 4x^3 - 6$

i)

ii)

iii)

90. $f'(x) = x^2(2-x)^3(x+3)$

i)

ii)

iii)

91. $f'(x) = \frac{x}{x^2-4}$

i)

ii)

iii)

92. $f'(x) = \frac{x^2}{x^2-4}$

i)

ii)

iii)

93. $f'(x) = \frac{x^3+8}{x^2-4}$

i)

ii)

iii)

94. $f'(x) = \frac{x}{x^2+4}$

i)

ii)

iii)

ANSWERS:

1) A	6) D	11) D	16) C	21) D	26) E	31) y =1/2	36) D	41) 3	46) C
2) D	7) D	12) A and B	17) A	22) D	27) B	32) y = 0	37) D	42) 6	47) E
3) E	8) C	13) C	18) D	23) E	28) E [D]	33) No HA	38) B	43) 8	
4) C	9) E	14) A	19) C	24) B	29) C	34) E	39) B	44) A	
5) B	10) D	15) B	20) B	25) D	30) C	35) D	40) E	45) C	

76) E	81) B	86) B	89) max at x = -0.956, min at x = 1.724, POI at x = 1
77) D	82) D	87) E	90) min at x = -3, neither at x = 0, max at x = 2, min at , POI at x = -2.351, 0, and 0.851
78) B	83) E	88) A	91) neither at x = ± 2, max at x = 0, No POIs
79) D	84) B		92) neither at x = ± 2 and at x = 0, POI at x = 0
80) B	85) C		93) neither at x = ± 2, POI at x = 0 and x = 4
			94) min at x = 0, POI at x = ± 2