Part A

1. Find the slope of \( f(x) \) at \( x = 0 \) for \( f(x) = (x+1)\ln(2e^{-0.5x} - 3x - 1) \).

**REMEMBER**...the derivative at a point, \( f'(x) \), slope of \( f(x) \), slope of tangent line, and instantaneous rate of change all mean the same thing. You just do different things depending if you have a function (use the formulas), a table (difference quotient), or a graph (find the slope by estimating rise over run).

<table>
<thead>
<tr>
<th></th>
<th>(A) (-\ln 2)</th>
<th>(B) (\ln 4)</th>
<th>(C) (4)</th>
<th>(D) (-4)</th>
<th>(E) (2)</th>
</tr>
</thead>
</table>

2. Find the slope of the secant line of \( f(x) \) from \( x = -1 \) to \( x = 0 \) for \( f(x) = \arcsin x \).

**REMEMBER**...the derivative over an interval, slope of secant line, average rate of change, and difference quotient all mean the same thing. You always do a slope formula \( \frac{y_2 - y_1}{x_2 - x_1} \) of \( \frac{f(b) - f(a)}{b - a} \).

<table>
<thead>
<tr>
<th></th>
<th>(A) (-\frac{\pi}{2})</th>
<th>(B) (\frac{\pi}{2})</th>
<th>(C) (0)</th>
<th>(D) (-\frac{\pi}{4})</th>
<th>(E) (\pi)</th>
</tr>
</thead>
</table>

3. \( \lim_{x \to \infty} \frac{x(4x^2 - 1)(x - 9)}{(x^3 - x)(4 - x^2)} = \)

<table>
<thead>
<tr>
<th></th>
<th>(A) (-\infty)</th>
<th>(B) (4)</th>
<th>(C) (-4)</th>
<th>(D) (0)</th>
<th>(E) (+\infty)</th>
</tr>
</thead>
</table>
Questions #4 to #7 refer to the following situation.

A bug begins to crawl up a vertical wire at time $t = 0$. The velocity $v$ of the bug at time, $t$, $0 \leq t \leq 8$, is given by the function whose graph is shown above.

4. At what value of $t$ does the bug change direction?

   (A) 2  (B) 4  (C) 6  (D) 7  (E) 8

5. What is the position at $t = 8$?

   (A) 14  (B) 13  (C) 11  (D) 8  (E) 6

6. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

   (A) 14  (B) 13  (C) 11  (D) 8  (E) 6

7. What is the average velocity of the bug from $t = 0$ to $t = 8$?

   (A) $\frac{7}{4}$  (B) $\frac{13}{8}$  (C) $\frac{11}{8}$  (D) 1  (E) $\frac{3}{4}$
8. Identify the points where \( f(x) \) is not differentiable.

\[
f(x) = \begin{cases} 
-(x+4)^2+1, & x \leq -4 \\
1, & -4 < x < -2 \\
2, & x = -2 \\
1, & -2 < x \leq 0 \\
e^x, & 0 < x < 4 \\
\sqrt[3]{x-6}, & x > 4 
\end{cases}
\]

(A) -2 and 4 only  
(B) -4, -2, and 4 only  
(C) -2, 0, and 4 only  
(D) -2, 0, 4, and 6 only  
(E) -4, -2, 0, 4, and 6

9. If \( f \) is continuous of the closed interval \([1, 3]\) with \( f'(x) < 0 \) on the open interval \((1, 3)\), then

(A) \( f(x) \) does not have a minimum on \([1, 3]\)  
(B) \( f(x) \) does not have a maximum on \([1, 3]\)  
(C) \( f(1) \geq f(x) \) for all \( x \) on the closed interval \([1, 3]\)  
(D) \( f(1) \leq f(x) \) for all \( x \) on the closed interval \([1, 3]\)  
(E) \( f(3) \geq f(x) \) for all \( x \) on the closed interval \([1, 3]\)

10. Use the graph of \( f(x) \) in the figure shown above to evaluate \( \int_{-4}^{10} f(x) \, dx \).

(A) 12  
(B) 13  
(C) 15  
(D) 16.5  
(E) 20.5
11. The graph of $f'$, the derivative of the function $f$, is shown above. Which of the following statements is **FALSE** about $f$?

(A) $f$ is concave down for $-1 \leq x \leq 1$
(B) $f$ is decreasing for $0 \leq x \leq 2$
(C) $f$ is increasing for $-2 \leq x \leq 0$
(D) $f$ has a maximum at $x = 0$
(E) $f$ has a point of inflection at $x = -1$ and $x = 1$

---

Questions #12 and #13 refer to the following piece-wise function.

Let $f(x) = \begin{cases} \sin \pi x & \text{for } 0 < x \leq 1 \\ \ln x^2 & \text{for } 1 < x \leq \pi \end{cases}$

12. $\lim_{x \to 1} f(x)$ is

   (A) 0  (B) 1  (C) -1  (D) $\pi$  (E) nonexistent

---

13. $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$ is

   (A) 0  (B) $\pi$  (C) -1  (D) 2  (E) nonexistent
14. If \( y = \frac{4 - 5x^2}{8x + 9} \), then \( \frac{dy}{dx} = \)

(A) \( \frac{-2(20x^2 - 25x - 8)}{(8x + 9)} \)  
(B) \( \frac{-2(20x^2 + 45x + 16)}{(8x + 9)^2} \)  
(C) \( \frac{2(10x^2 + 25x + 8)}{(8x + 9)^2} \)  
(D) \( \frac{-2(20x^2 + 45x)}{(8x + 9)^2} \)  
(E) \( \frac{2(20x^2 + 25x + 16)}{(8x + 9)^2} \)

15. The line tangent to the graph of \( y = xe^x + e^x \) at \( x = 0 \) intersects the x-axis at \( x = \)

(A) -1  
(B) \( \frac{-1}{2} \)  
(C) \( \frac{1}{2} \)  
(D) 1  
(E) 2
16. If \( \int_a^b f(x) \, dx = a + 2b \), then \( \int_a^b (f(x) + 5) \, dx = \)

(A) \( a + 2b + 5 \)  \quad (B) \( 5b - 5a \)  \quad (C) \( 7b - 4a \)  \quad (D) \( 7b - 5a \)  \quad (E) \( 7b - 6a \)

Questions #17 and #18 refer to the following graph.

>The graph of \( f'' \) is shown above.

17. For what values of \( x \) does the graph of \( f \) concaves down?

(A) \( (0, m) \) only  \quad (B) \( (j, \infty) \) only  \quad (C) \( (-\infty, a) \) and \( (k, \infty) \)

(D) \( (0, m) \) and \( (j, \infty) \)  \quad (E) \( (k, j) \) and \( (j, \infty) \)

18. For what values of \( x \) does the graph of \( f \) have a point of inflection?

(A) \( k \) and \( b \) only  \quad (B) \( 0 \) and \( m \) only  \quad (C) \( a, k \) and \( j \)

(D) \( 0, m, \) and \( j \)  \quad (E) \( a \) and \( k \) only
The first derivative of the function $f$ is given by $f'(x) = -(x-a)(x-b)^2$. The graph of $f'$ is shown above.

19. What values of $x$ are the critical points of $f$?

(A) $a$ and $k$ only  (B) $a$, $k$, and $j$  (C) $0$ and $m$ only  (D) $0$, $m$, and $j$  (E) $k$ and $b$ only

20. For what values of $x$ does the graph of $f$ has a local maximum?

(A) $a$ only  (B) $0$ only  (C) $k$ only  (D) $m$ only  (E) $0$ and $j$ only

21. For what values of $x$ does the graph of $f$ concaves down?

(A) $(0, m)$ only  (B) $(j, \infty)$ only  (C) $(k, \infty)$ only

(D) $(0, m)$ and $(j, \infty)$  (E) $(k, j)$ and $(j, \infty)$

22. For what values of $x$ does the graph of $f$ have a point of inflection?

(A) $k$ and $b$ only  (B) $0$ and $m$ only  (C) $a$, $k$ and $j$  (D) $0$, $m$, and $j$  (E) $a$ and $k$ only
23. The derivative, \( g' \), of a function is continuous and has exactly two zeros. Selected values of \( g' \) are given in the table above. If the domain of \( g \) is the set of all real numbers, then \( g \) has a local minimum at which values of \( x \)?

(A) \(-2\) only  
(B) \(2\) only  
(C) \(-2\) and \(2\) only  
(D) \(0\) only  
(E) no such values

24. The 2nd derivative, \( g'' \), of a function is continuous and has exactly two zeros. Selected values of \( g'' \) are given in the table above. If the domain of \( g \) is the set of all real numbers, then \( g \) is concaving down on which of the following intervals?

(A) \(-2 \leq x \leq 2\) only  
(B) \(x \leq -2\) or \(x \geq 2\)  
(C) \(x \geq -2\) only  
(D) \(x \geq 2\) only  
(E) \(-1 \leq x \leq 1\) only

25. The rate at which a faucet pours water into a tank is shown in the graph above. If the tank has 10 gallons at \( t = 0 \) minutes, then to the nearest whole number, the amount of water in the tank at 20 minutes is about

(A) 10 gallons  
(B) 20 gallons  
(C) 30 gallons  
(D) 40 gallons  
(E) 50 gallons
26. The graph of $f'$, the derivative of $f$, is the line shown in the figure above. If $f(0) = 5$, then $f(4) =$

(A) 13  
(B) 17  
(C) 20  
(D) 21  
(E) 23

27. Consider the curve given by $x^2 + y = 1 + 2x(y - 1)^2$. Which of the following statements is true?

(A) The curve concaves down at $(1,2)$ since $\frac{d^2y}{dx^2}(1,2) = \frac{2}{3}$.

(B) The curve concaves up at $(1,2)$ since $\frac{d^2y}{dx^2}(1,2) = \frac{2}{3}$.

(C) The curve concaves down at $(1,2)$ since $\frac{d^2y}{dx^2}(1,2) = -\frac{5}{8}$.

(D) The curve concaves up at $(1,2)$ since $\frac{d^2y}{dx^2}(1,2) = -\frac{5}{8}$.

(E) The curve is linear at $(1,2)$ since $\frac{d^2y}{dx^2}(1,2) = 0$. 
28. The graph of the function \( f \) is given below. Which of these graphs could be the derivative of \( f \)?
29. The graph of the function $f$ is given below. Which of these graphs could be the derivative of $f$?
30. The graph of the function $f$ is given below. Which of these graphs could be the derivative of $f$?

(A) 

(B) 

(C) 

(D) 

(E)
Questions #31 to #34 refer to the following function.

Let \( f \) be the function with derivative given by \( f'(x) = x - \frac{3}{x^2} \).

31. What values of \( x \) are the critical points of \( f \)?

(A) 0 only
(B) 0 and \( \sqrt[3]{3} \)
(C) \( \sqrt[3]{3} \) only
(D) \( \sqrt[3]{-6} \) only
(E) 0 and \( \sqrt[3]{-6} \)

32. On which of the following intervals is \( f \) decreasing?

(A) \((-\infty, \sqrt[3]{3})\) only
(B) \((-\infty, 0)\) or \((0, \sqrt[3]{3})\)
(C) \((0, \sqrt[3]{3})\) only
(D) \((-\infty, 0)\)
(E) \((\sqrt[3]{3}, \infty)\) only

33. On which of the following intervals is \( f \) concave down?

(A) \((-\infty, \sqrt[3]{-6})\) only
(B) \((-\infty, 0)\) or \((\sqrt[3]{-6}, \infty)\)
(C) \((\sqrt[3]{-6}, 0)\) only
(D) \((-\infty, 0)\)
(E) \((0, \infty)\) only

34. For what values of \( x \) does the graph of \( f \) have a point of inflection?

(A) \( \sqrt[3]{6} \) only
(B) 0 only
(C) \( \sqrt[3]{-6} \) and 0
(D) 1 only
(E) there are no points of inflection
35. If \( f(x) = e^{-x} + 3x \), then the critical points of \( f \) are

(A) \( x = 0 \) only

(B) \( x = -\ln 3 \) only

(C) \( x = \ln 3 \) only

(D) \( x = -\ln 3 \) and \( x = \ln 3 \)

(E) there are no critical points

36. If \( y = \cos^3 x \sin 3x \), then \( \frac{dy}{dx} = \)

(A) \( \cos^2 x (3 \cos x \cos 3x + \sin x \sin 3x) \)

(B) \( \cos^2 x (\sin x \sin 3x - 3 \cos x \cos 3x) \)

(C) \( 3 \cos^2 x (\cos x \cos 3x - \sin x \sin 3x) \)

(D) \( 3 \cos^2 x (\cos 3x - \sin 3x) \)

(E) \( 3 \cos^2 x (\sin 3x - \cos 3x) \)

37. \( \int_{-1}^{2} x^3 - 2x + 5 \, dx = \)

(A) \( -\frac{33}{4} \)

(B) \( -\frac{1}{4} \)

(C) \( \frac{57}{4} \)

(D) \( \frac{63}{4} \)

(E) \( \frac{87}{4} \)
38. \[ \int_{-1}^{\sqrt{3}} \frac{1}{1 + x^2} \, dx = \]

| (A) | $\frac{5\pi}{12}$ |
| (B) | $\frac{7\pi}{12}$ |
| (C) | $\frac{5\pi}{6}$ |
| (D) | $\frac{7\pi}{6}$ |
| (E) | $\frac{5\pi}{4}$ |

39. The radius of a circle is increasing at a constant rate of 0.25 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is $10\pi$ meters?

(A) $0.25\pi$ m²/sec  
(B) $2.5\pi$ m²/sec  
(C) $5\pi$ m²/sec  
(D) $10\pi$ m²/sec  
(E) $12.5\pi$ m²/sec

Hint: \[ A = \pi r^2 \]
\[ C = 2\pi r \]
Questions #40 to #42 refer to the following table.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

40. Find the instantaneous rate of change of $f(x)$ at $x = 3$.

(A) $\frac{12}{5}$  (B) 4  (C) $\frac{6}{5}$  (D) 3  (E) $\frac{16}{5}$

41. Find the average rate of change of $f(x)$ on $[0, 7]$.

(A) $\frac{6}{7}$  (B) 1  (C) $\frac{8}{7}$  (D) $\frac{9}{7}$  (E) $\frac{10}{7}$

42. Use the trapezoidal rule to find the average value of $f(x)$ on $[0, 7]$.

(A) $\frac{43}{7}$  (B) 6  (C) $\frac{45}{7}$  (D) 5  (E) $\frac{48}{7}$
43. If \( y = \cos^2(1 - 2x^4) \), then \( \frac{dy}{dx}_{x=-1} = \)

(A) \( 8\cos(1) \)  
(D) \( -8\cos(1)\sin(1) \)

(B) \( -16\sin(1) \)  
(E) \( 16\cos(1)\sin(1) \)

(C) \( 16\sin(1) \)  

44. If \( 3p + 2q = 600 \), the maximum value of \( p \cdot q \) is

(A) 100  
(B) 150  
(C) 600  
(D) 15,000  
(E) 60,000
Part A

45. The function \( f(x) \) is continuous for the closed interval \([a, b]\) and differentiable for the open interval \((a, b)\). If \( f(a) = f(b) \) and \( f'(x) \) changes sign only once on \([a, b]\), then which of the following could be false?

(A) \( \lim_{x \to c} f(x) = f(c) \) for all values \( c \) such that \( a < c < b \).

(B) \( f \) has a minimum or maximum value on \( a \leq x \leq b \).

(C) \( f \) has a point of inflection on \( a \leq x \leq b \).

(D) \( f'(c) = \frac{f(b) - f(a)}{b - a} \) for some \( c \) such that \( a < c < b \).

(E) \( f'(c) = 0 \) for some \( c \) such that \( a < c < b \).

46. Let \( f \) be continuous for \( 0 \leq x \leq 5 \) where \( (0, 13) \) and \( (5, 3) \) are the endpoints of \( f \). The Intermediate Value Theorem guarantees which of the following?

(A) \( f(c) = 2 \) for some \( c \) such that \( 0 < c < 5 \).

(B) \( f'(c) = 2 \) for some \( c \) such that \( 0 < c < 5 \).

(C) \( f'(c) = 0 \) for some \( c \) such that \( 0 < c < 5 \).

(D) \( f(c) = 4 \) for some \( c \) such that \( 0 < c < 5 \).

(E) \( \lim_{x \to c} f(x) = f(c) \) for all values \( c \) on \( 0 < c < 5 \).

47. If \( f \) is continuous for \( 0 \leq x \leq 6 \) and \( f(0) = 1 \) and \( f(6) = 7 \), then which of the following could be false?

(A) \( f \) has no vertical asymptotes on \( 0 \leq x \leq 6 \).

(B) There exists a value \( c \) on \( 0 < c < 6 \) such that the slope of the tangent line at \( x = c \) is 1.

(C) \( f(c) = 2 \) for some \( c \) such that \( 0 < c < 6 \).

(D) \( f(c) = 0 \) for some \( c \) such that \( 0 < c < 6 \).

(E) \( \lim_{x \to c} f(x) \) exists for all values \( c \) on \( 0 < c < 6 \).
Questions #48 to #55 refer to the following graph.

True or False.

Use the graph of $f(x)$ above to determine if the following statements are true or false.

48. $f(x)$ has an absolute minimum at $x = 3$.

49. There exists a value $c$ such that $f(c) \geq f(x)$.

50. $\lim_{x \to -\infty} f(x) = 1$

51. $\lim_{x \to 1} f(x) = f(1)$

52. $\lim_{x \to 2} f(x)$ exists.

53. $\lim_{h \to 0} \frac{f(3 + h) - f(3)}{h}$ exists.

54. There exists a value $c$ on $1 < c < 2$ such that $\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = 0$.

55. $\lim_{x \to -\infty} f'(x) = 0$
76. A particle moves along a straight path with the acceleration $a(t) = \tan^{-1}(e^t + 1)$. If the velocity at $t = 7$ is 18 meters/sec then find the velocity at $t = 10$.

(A) 20.005 m/s  (B) 21.205 m/s  (C) 22.712 m/s  (D) 23.084 m/s  (E) 24.030 m/s

Questions #77 and #78 refer to the following situation.

A particle moves along a straight path with the velocity $v(t) = (2t - 1)^2 \cos t$.

77. How many times is the particle at rest on the interval $0 \leq t \leq 5$?

**REMEMBER**...it doesn't matter if you're finding when the particle is at rest, when the velocity is zero, changes direction, relative extrema, critical points, or when the particle is farthest left or right. You always do the same thing - (1) first set $v(t)$ or $f'(t)$ equal to zero, then (2) solve OR graph $v(t)$ or $f'(t)$ and find the $x$-intercepts, then (3) answer the question appropriately.

(A) One  (B) Two  (C) Three  (D) Four  (E) Five

78. How many times is the particle at a relative extrema on the interval $0 \leq t \leq 5$?

(A) One  (B) Two  (C) Three  (D) Four  (E) Five
79. Consider \( f(x) = 4x - \tan x \) and \( g(x) = \sin^{-1} x \) for \( 0 < x < \frac{\pi}{2} \). Which of the following is the equation of the line tangent to the graph of \( f(x) \) at the point where \( f(x) \) and \( g(x) \) have parallel tangent lines?

(A) \( y = 1.799x + 0.236 \)
(B) \( y = 1.799x + 0.734 \)
(C) \( y = 1.799x + 1.045 \)
(D) \( y = 2.229x + 0.831 \)
(E) \( y = 2.229x + 1.190 \)

80. The foot of a 20 ft ladder is being pulled away from a wall at the rate of 1.5 ft/sec. At the instant when the foot is 12 ft away from the wall, the angle the ladder makes with the floor is decreasing at the rate (in radians/sec) of:

(A) \( \frac{3}{50} \)  
(B) \( \frac{1}{16} \)  
(C) \( \frac{3}{40} \)  
(D) \( \frac{1}{8} \)  
(E) \( \frac{3}{32} \)
81. As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area $400\pi$ square feet. The depth $h$, in feet, of the water in the conical tank is changing at the rate of $(h - 12)$ feet per minute. At what rate (in feet per minute) is the depth of the water in the cylindrical tank changing when the depth of the conical tank is 3 feet (i.e., $h = 3$)?

(Note: Volume of a cylinder is $V = \pi r^2 h$ and volume of a cone is $V = \frac{1}{3} \pi r^2 h$.)

(A) \(\frac{9}{400}\)  (B) \(\frac{9}{200}\)  (C) \(\frac{3}{400}\)  (D) \(\frac{3}{200}\)  (E) \(\frac{3}{100}\)
82. \( f(t) \) has how many **relative extrema** if \( f'(t) = (e^t - 1)(t - 2)^3(t + 3)^2 \) the particle experience on the interval \(-5 \leq t \leq 5\)?

(A) One  (B) Two  (C) Three  (D) Four  (E) Five

---

**Questions #83 and #84 refer to the following situation.**

The velocity of a bug moving in a straight line given by \( v(t) = \sqrt{t} - 1 \) ft/sec.

83. How far from starting point from \( t = 0 \) to \( t = 3 \) seconds?

(A) 0.186 ft  (B) 0.345 ft  (C) 0.464 ft  (D) 0.543 ft  (E) 0.720 ft

---

84. What is the distance traveled from \( t = 0 \) to \( t = 3 \) seconds?

(A) 1.131 ft  (B) 1.246 ft  (C) 1.372 ft  (D) 1.484 ft  (E) 1.597 ft
Questions #85 to #87 refer to the following graph.

85. The regions A, B, and C in the figure above are bounded by the graph of the function $f$ and the x-axis. If the area of each region is 4, then the value of $\int_{-3}^{3} f(x) \, dx =$

(A) -4  (B) -2  (C) 0  (D) 4  (E) 8

86. The value of $\int_{-3}^{5} f(x + 2) \, dx =$

(A) -4  (B) -2  (C) 0  (D) 4  (E) 8

87. The value of $\int_{-3}^{3} (f(x) + 2) \, dx =$

(A) -4  (B) -2  (C) 0  (D) 4  (E) 8
88. Use the trapezoidal approximation for find the average value of the twice differentiable function, $f$, represented by the table above, on the closed interval $[0, 8]$?

<table>
<thead>
<tr>
<th>$x$</th>
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<th>6</th>
<th>8</th>
</tr>
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<tbody>
<tr>
<td>$f(x)$</td>
<td>50</td>
<td>40</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

(A) 25.5  (B) 26  (C) 26.5  (D) 27  (E) 27.5

89. Let $f(x) = 2x^3 + x^2$, where both $f$ and $f^{-1}$ (the inverse of $f$) exist, are continuous, and differentiable for $x > 0$. $\frac{d}{dx} \left( f^{-1}(x) \right)$ at $x = 1$ is

(A) 0.125  (B) 0.256  (C) 0.336  (D) $\frac{1}{4}$  (E) $\frac{3}{8}$
90. If $a \neq 0$, then $\lim_{x \to a} \frac{a^3 - x^3}{x^4 - a^4}$ is

\[
\begin{array}{cc}
(A) & -\frac{3}{2a^2} \\
(B) & -\frac{1}{2a^2} \\
(C) & -\frac{3}{4a} \\
(D) & -\frac{2}{5a} \\
(E) & \text{nonexistence}
\end{array}
\]

91. $\lim_{x \to a} \frac{a - x}{e^x}$ is

\[
\begin{array}{cc}
(A) & 0 \\
(B) & e^a \\
(C) & 1 \\
(D) & -1 \\
(E) & \text{nonexistence}
\end{array}
\]

92. $\lim_{x \to a} \frac{e^x}{a - x}$ is

\[
\begin{array}{cc}
(A) & 0 \\
(B) & e^a \\
(C) & 1 \\
(D) & -1 \\
(E) & \text{nonexistence}
\end{array}
\]
93. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\sqrt{x^2 + 7} & 0 \leq x < 3 \\
7 - x & 3 \leq x \leq 5.
\end{cases}
\]
Which of the following is false?

(A) \( f \) is continuous but not differentiable at \( x = 3 \).

(B) \( f'(2) = 0.603 \).

(C) \( \lim_{x \to 3} f(x) = f(3) \).

(D) The average value of \( f(x) \) on the closed interval \( 0 \leq x \leq 5 \) is 3.081.

(E) \( \frac{1}{5} \int_0^5 f(x) \, dx = 6.135 \).

94. What is the average velocity, in ft/sec, of a particle moving a straight path from time \( t = 0 \) to time \( t = 2 \) if the velocity function is \( v(t) = t^2 e^t + t e^t \)?

(A) 7.086 ft/sec  \hspace{1cm} (B) 9.447 ft/sec  \hspace{1cm} (C) 10.584 ft/sec  \hspace{1cm} (D) 11.924 ft/sec  \hspace{1cm} (E) 13.342 ft/sec

95. A particle moves along the y-axis so that at any time \( t > 0 \), its velocity is given by \( v(t) = -3.4 \sin(0.2t - \pi) \). If the position of the particle is at \( y = 6 \) at time \( t = 3 \), then find the initial position of the particle.

(A) 3.031 \hspace{1cm} (B) 3.234 \hspace{1cm} (C) 3.509 \hspace{1cm} (D) 3.750 \hspace{1cm} (E) 3.983
96. \[ \int \frac{x^3 - 1 + \sqrt{x}}{x^4} \, dx = \]

(A) \( \frac{1}{x^2} + \frac{1}{4x^3} - \frac{1}{3\sqrt{x}^3} + C \)  
(B) \( -\frac{1}{x} - \frac{1}{4x^3} - \frac{1}{3\sqrt{x}^3} + C \)  
(C) \( \ln|x| + \frac{1}{4x^3} - \frac{2}{5\sqrt{x}^5} + C \)  
(D) \( \ln x + \frac{1}{3x^3} - \frac{2}{5\sqrt{x}^5} + C \)  
(E) \( \ln|x| + \frac{1}{3x^3} - \frac{2}{5\sqrt{x}^5} + C \)

97. A cold ham, having the temperature of 42°F (Fahrenheit), is heated in an oven set to 425°F. The temperature of the ham is changing at a rate of \( 134.05 e^{-0.35t} \) degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the ham at 15 minutes?

(A) 412°  
(B) 416°  
(C) 419°  
(D) 423°  
(E) 425°

98. The rate at which water is filling and draining (positive rate is filling and negative rate is draining) in a tank during a 10 minute period is given by \( r(t) = 0.25t^3 - 3.25t^2 + 10t \) gal/min. In the 10 minute period, what is the total amount of water that filled the tank during the time the water level was rising?

(A) 32.238gal  
(B) 36.085 gal  
(C) 41.667 gal  
(D) 48.979 gal  
(E) 56.292 gal
99. Suppose \( f(x) \) is continuous and \( f(x) \geq 0 \) for all real numbers. If the average value of the function \( f(x) \) on the closed interval \([-1, 4]\) is 3, then the area bounded by \( f(x) \) and the \( x \)-axis from \( x = -1 \) to \( x = 4 \) is

(A) 9  (B) 12  (C) 15  (D) 18  (E) cannot determine

100. The velocity of a particle is given by \( v(t) = t^4 - 4t^3 \). Which of the following statements is true?

(A) \( v(3) > 0 \) and \( a(3) < 0 \); therefore the speed of the particle is decreasing at \( t = 3 \).

(B) \( v(3) < 0 \) and \( a(3) < 0 \); therefore the speed of the particle is increasing at \( t = 3 \).

(C) \( v(3) < 0 \) and \( a(3) > 0 \); therefore the speed of the particle is decreasing at \( t = 3 \).

(D) \( v(3) < 0 \) and \( a(3) > 0 \); therefore the speed of the particle is increasing at \( t = 3 \).

(E) \( v(3) > 0 \) and \( a(3) > 0 \); therefore the speed of the particle is increasing at \( t = 3 \).

101. Suppose \( f \) is an even function and \( f = F'(t) \). If \( \int_{0}^{2} f(t) \, dt = 5 \) and \( F(2) = 3 \), then \( F(-2) = \)

(A) \(-7\)  (B) \(-2\)  (C) \(3\)  (D) \(8\)  (E) \(13\)
102. Sketch a graph of $y = f(x)$ on the interval $0 \leq x \leq 4$ whose trapezoidal sum underapproximates $\int_0^4 f(x) \, dx$, and a right Riemann sum also underapproximates $\int_0^4 f(x) \, dx$.

103. The rate at which people enter an amusement park on a given day is modeled by the function $E$ defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$ 

The rate at which people leave the amusement park on the same day is modeled by the function $L$ defined by

$$L(t) = \frac{9892}{(t^2 - 38t + 370)}.$$ 

Both $E(t)$ and $L(t)$ are measured in people per hour and time $t$ is measured in hours after midnight. The park is open from 9 am to 11 pm ($9 \leq t \leq 23$). At 9 am, there are no people in the park.

(a) To the nearest whole number, how many people have entered the park by 5:00 pm ($t = 17$)?

(b) The price of admission to the park is $15 until 5:00$ pm ($t = 17$). After 5:00 pm, the price of admission to the park is $11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) At what time $t$, for $9 \leq t \leq 23$, does the model predict that the number of people in the park is a maximum? To the nearest whole number, state what that maximum number is.
### Answers

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103 a) \[\int_{t_1}^{t_2} E(t)dt = 6004\]  \[15 \int_{t_1}^{t_2} E(t)dt + 11\] \[\int_{t_1}^{t_2} E(t)dt = $104,048\]

b) \[\int_{t_1}^{t_2} E(t)dt = 6004\]  \[15 \int_{t_1}^{t_2} E(t)dt + 11\] \[\int_{t_1}^{t_2} E(t)dt = $104,048\]

c) Total Number of people = Number of people Entering − Number of people Leaving

\(p(t) = e(t) − l(t)\)

Maximize \(p(t)\) by taking the derivative and setting \(p'(t) = 0\),

\[p'(t) = e'(t) − l'(t) = 0\]

Since \(e'(t)\) is really \(E(t)\) and \(l'(t)\) is really \(L(t)\), we have

\[E(t) − L(t) = 0\]

Therefore, at \(t = 15.794\) hours there is a maximum of about 3725 people.