For problems #1 and #2, find the value of \( c \) guaranteed by the Mean Value Theorem on the indicated interval such that
\[
f'(c) = \frac{f(b) - f(a)}{b - a}.
\]

1. \( f(x) = x^2 \) on \([1, 3]\)

2. \( f(x) = x^3 + 1 \) on \([-1, 1]\)

3. Which of the following satisfy the hypotheses of Mean Value Theorem on the interval \([0, 2]\)?

I. \( f(x) = \sin \pi x + \cos 2x \)

II. \( f(x) = \sqrt[3]{x - 2} \)

III. \( f(x) = |x^2 - 2x| \)

(A) I only  (B) II only  (C) III only  (D) I and II  (E) I, II, and III

4. Which of the following satisfy the hypotheses of Mean Value Theorem on the interval \([0, 3]\)?

Note: this is the same as #3, except the interval is different.

I. \( f(x) = \sin \pi x + \cos 2x \)

II. \( f(x) = \sqrt[3]{x - 2} \)

III. \( f(x) = |x^2 - 2x| \)

(A) I only  (B) II only  (C) III only  (D) I and II  (E) I, II, and III
5. Which of the following satisfy the hypotheses of Rolle’s Theorem on the interval \([0,2]\)?

I. \(f(x) = \frac{1}{|x-1|}\)
II. \(f(x) = |x - 1|\)
III. \(f(x) = x^2 - 2x\)
IV. \(f(x) = \sin 2x\)

(A) I only  (B) II only  (C) III only  (D) IV only  (E) I and II

6. Let \(f\) be continuous for \(0 \leq x \leq 5\) where \((0,13)\) and \((5,3)\) are the endpoints of \(f\). The Intermediate Value Theorem guarantees which of the following?

(A) \(f(c) = 2\) for some \(c\) such that \(0 < c < 5\).

(B) \(f''(c) = 2\) for some \(c\) such that \(0 < c < 5\).

(C) \(f''(c) = 0\) for some \(c\) such that \(0 < c < 5\).

(D) \(f(c) = 4\) for some \(c\) such that \(0 < c < 5\).

(E) \(\lim_{x \to c} f(x) = f(c)\) for all values \(c\) on \(0 < c < 5\).

7. How many values of \(c\) are guaranteed by Rolle’s Theorem for \(f(x)\) below on the interval \([-10,10]\)?

If \(f(x) = \begin{cases} \frac{\sin x}{x}, & \text{for } x \neq 0 \\ 1, & \text{for } x = 0 \end{cases}\)

(A) 4  (B) 5  (C) 6  (D) 7  (E) the theorem does not apply
8. Suppose $f$ is continuous for $0 \leq x \leq 6$ and differentiable for $0 < x < 6$. If $f(0) = 1$ and $f(6) = 7$, then which of the following could be false?

(A) $f$ has no vertical asymptotes on $0 \leq x \leq 6$.

(B) There exists a value $c$ on $0 < c < 6$ such that the slope of the tangent line at $x = c$ is 1.

(C) $f(c) = 2$ for some $c$ such that $0 < c < 6$.

(D) $f(c) = 0$ for some $c$ such that $0 < c < 6$.

(E) $\lim_{x \to c} f(x)$ exists for all values $c$ on $0 < c < 6$. 
ANSWERS:

1) $2 \quad 4) \ A \quad 7) \ B$
2) $\pm \frac{\sqrt{3}}{3} \quad 5) \ C \quad 8) \ D$
3) $E \quad 6) \ D$