Chapter 3 TEST Day 1 Study Session

1. Find POIs.

2. If $f'$ is given, find POIs.
Evaluate the limit:

\( \lim_{x \to \infty} \frac{x^3 + 5x^3}{8x^3 - x} \)

\( \lim_{x \to \infty} \frac{x^2 + 5x}{8x^3 - x} \)

\( \lim_{x \to \infty} \frac{x^3 + 5x^3}{8x^2 - x} \)
True or False.

a) \(-2 < t < 0\) \(f\) increases
b) \(-3 < t < -2\) \(f\) decreases
c) \(0 < t < 3\)

d) \(f'\) is undefined at \(x = -1\) \(\&\) \(x = 1\)
e) \(f''\) is undefined at \(x = -1\) \(\&\) \(x = 1\)
f) \(f\) local max at \(x = 0\)
g) \(f\) local min at \(x = -2\)
h) \(f\) is linear on \(1 < x < 3\).

*Go over #43 on ch3 pt 7.*
Things to know about limits

Removable

at a & b, \( \lim_{x \to a} f \) & \( \lim_{x \to b} f \) exists

Essential
Jump Break

at a, b, & c, limit does not exist.

Not Differentiable at
- any discontinuity
- sharp corner
- cusp
- vertical tangents

\[ f = \sqrt[3]{x} \]
<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>$k$</td>
</tr>
</tbody>
</table>

If $f'(x) > 0$, and $f''(x) < 0$, find possible values of $k$. 
6. \begin{array}{c|ccccccc}
 x & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
 f'(x) & 5 & -3 & 0 & 2 & 1 & 0 & 2 \\
\end{array}

If $f'(x)$ has exactly 2 zeros, where is $f$ increasing?
7. \[ f(x) = \begin{cases} 
\cos x & x \leq 0 \\
1 & x > 0 
\end{cases} \]

True or False.

a) \( \lim_{x \to 0} f(x) = 1 \)

b) \( f(x) \) is differentiable at \( x = 0 \)

c) \( \lim_{x \to 0} f(x) = f(0) \)

d) \( \lim_{h \to 0} \frac{f(h+0) - f(0)}{h} \) exists
\[ x(t) = \frac{1}{2} t^3 - 4t^2 - 20t \]

is the position of a particle moving horizontally. When is the particle at rest?
9. Find when \( f(x) \) increases if \( f'(x) = x^3 - \frac{1}{x^2} \).
Find the signs of $f$, $f'$, and $f''$ at $1$. 
Find \( \frac{dy}{dx} \bigg|_{x=3} \) for

\[ x^3 + x^2 y = e \]
12. Write an equation where the rate of change of \( y \) is directly proportional to \( x \) and inversely proportional to the cube of \( z \).
13) At what value(s) does $y = x^3 - x^2$ have a point of inflection?

14) If $x(t) = t^4 - 2t^3$, then when is particle's velocity 0?
\[ 15 \lim_{h \to 0} \frac{(2+h)^3 - 2^3}{h} = \]

(a) Equate the Equation

(b) L'Hôpital's Rule
\[ \lim_{{h \to 0}} \frac{\sin\left(\frac{3\pi}{4} + h\right) - \sin \frac{3\pi}{4}}{h} \]

(a) Equate the equation.

(b) L'Hopital's Rule
If \( f(x) = x + e^{2x} \), find the equation of the tangent line at \( x = 0.5 \).
18. Find $f''(x)$ for $f(x) = \tan(e^{\sin x})$.

$$f'(x) = \text{...}$$

19. Find $f'(2)$.
20. Find where $f(x) = (x^3)(e^x)$ is concave down?
2. If \( f(x) = x^3 + 1 \) and \( g(x) = f^{-1}(x) \) and \( g(9) = 2 \), then find \( g'(9) \).
22. Find the signs of $f$, $f'$, $f''$. 

[Diagram of three graphs showing the signs of $f$, $f'$, and $f''$.]
23. Where does $y$ decrease if $y = \frac{1}{3}x^5 - 2x^3$?

24. Find the instantaneous rate of change of $f(x) = \frac{x - 5}{4 - x^2}$ at $x = 1$. 

JPF 24) Find the average rate of change of \( f(x) = \frac{x - 5}{y - x^3} \)
on \([0, 1] \).
25. \[ f(x) = \begin{cases} 
6x + 1 & x < 0 \\
2 & x = 0 \\
1 + x^2 & x > 0 
\end{cases} \]

**Graph**

**True or False**

\( \lim_{x \to 0} f(x) = 1 \)

\( \lim_{x \to 0} f(x) = f(0) \)

\( f'(0) \text{ exists} \)

\( f \text{ is differentiable at } x = 0 \)

\( \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \text{ exists} \)
\[ \begin{array}{c|cccc} x & 0 & 10 & 20 & 30 \\ \hline P(x) & 100 & 80 & 70 & 68 \end{array} \]

Find \( P'(10) = \)

Find \( P'(18) = \)

Find \( P'(30) = \)
27) Find the maximum velocity if the position is
\[ x(t) = 2t^3 - 8t^2 + t - 1000 \]
on \([0, 10]\).
\[ 28 \] Find \( \frac{dy}{d\theta} \) for \( y = \sin 2\theta \tan \theta \) at \( \theta = \frac{\pi}{6} \).
29) Know stuff about horizontal asymptote. Let \( y = k \) be H.A.

\[
\lim_{{x \to \infty}} y = k \quad \leftarrow \text{same same}
\]

\[\exists \text{ a value } x \quad \Rightarrow f(x) = k \text{.}\]

\(\text{ie} \) You can cross the H.A.
Find POI, if
\[ f''(x) = x^2(x-4)(x+3) \]