Obj:

to solve an **Implicitly written** differential equation for its **General Solution & Particular Solution**

We did this before except we did for an **Explicitly written** differential equation.

**Ex.** \( \frac{dy}{dx} = x^2 \)

Solve by finding the **Antiderivative**.

If \( \frac{dy}{dx} = x^2 \),

then \( y = \frac{1}{3}x^3 + C \).

This is the **General Soln.**

* Remember that there are an infinite number of **Antiderivatives**.
Ex: Find the particular soln of \( \frac{dy}{dx} = x^2 \) if \( y(1) = 3 \).

(This is also called an Initial Value problem.)

The particular soln is just one antiderivative that goes through \((1, 3)\).

If \( \frac{dy}{dx} = x^2 \), then

\[ y = \frac{1}{3} x^3 + C. \]

If \( y(1) = 3 \), then

\[ 3 = \frac{1}{3} (1)^3 + C \]
\[ \frac{8}{3} = C \]

\[ \therefore \text{ Particular Soln is} \]
\[ y = \frac{1}{3} x^3 + \frac{8}{3} \]
In 3-1, the differential equation will be implicitly written, mixed with \( x \) \& \( y \).

Example: \( \frac{dy}{dx} = x^2 y \).

We solve by the method of Separation of Variables.

\[
\frac{dy}{dx} = x^2 y
\]

Multiply over:

\( dy = x^2 y \, dx \)

Divide over:

\( \frac{1}{y} \, dy = x^2 \, dx \)

Integrate both sides:

\[
\int \frac{1}{y} \, dy = \int x^2 \, dx
\]

\( \ln |y| + C_1 = \frac{1}{3} x^3 + C_2 \)

\( -C_1 - C_1 \)

\( \ln |y| = \frac{1}{3} x^3 + C_2 - C_1 \)

This is also a constant

\( \therefore \ln |y| = \frac{1}{3} x^3 + C \)

General solution
Ex: Find the particular solution, 
\[ y = f(x), \text{ of } \frac{dy}{dx} = x^2 y \text{ if } \]
\[ y(2) = 1, \quad y \neq 0 \]

\[ \frac{dy}{dx} = x^2 y \]

Find the general soln
\[ \ln |y| = \frac{1}{3} x^3 + C \]

Now plug in initial value \( y(2) = 1 \).
\[ \ln |1| = \frac{1}{3} (2)^3 + C \]
\[ 0 = \frac{8}{3} + C \]
\[ -\frac{8}{3} = C \]

\[ \therefore \ln |y| = \frac{1}{3} x^3 - \frac{8}{3} \]

Now solve for \( y \).
\[ \ln |y| = \frac{1}{3} e^2 - \frac{8}{3} \]
\[ |y| = e^{\frac{1}{3} e^2 - \frac{8}{3}} \]
\[ |y| = e^{\frac{1}{3}e^3 - \frac{\pi}{3}} \]

by definition of Absolute Value, 

\[ y = \pm e^{\frac{1}{3}e^3 - \frac{\pi}{3}} \]

To satisfy the initial value of \( y(x) = 1 \), the particular soln must be the positive expression.

\[
\begin{align*}
\text{(since } & l = e^{\frac{1}{3}(x^3 - \frac{\pi}{3})} \\
\text{) } & l = e^0 \\
\text{) } & l = 1
\end{align*}
\]

\[ \therefore \text{ the particular soln, } y = f(x), \text{ is } y = e^{\frac{1}{3}e^3 - \frac{\pi}{3}} \]

You could also re-write the expression 

\[ y = e^{\frac{1}{3}e^3} e^{-\frac{\pi}{3}} \]

\[
\begin{align*}
\text{or } & y = e^\frac{\pi}{3} e^{\frac{1}{3}e^3} \\
\text{Either equation works!}
\end{align*}
\]