Part A

1. If \( f'(2) = 7 \) and \( f''(2) = 0 \), then which of the following must be true?

I. \( f(x) \) has a local extreme value when \( x = 2 \)
II. \( f(x) = 7 \) for all \( x \)
III. \( (2, 7) \) is a point of inflection

(A) None  (B) I only  (C) II only
(D) III only  (E) I and III only

2. If \( f \) is differentiable for all \( x \), and has a local maximum at \( x = 3 \), then which of the following must be true?

I. \( f'(3) = 0 \)
II. \( f''(3) < 0 \)
III. \( f \) is continuous at \( x = 3 \)

(A) I only  (B) II only  (C) I and II only
(D) I and III only  (E) I, II, and III

3. What are all the \( x \)-coordinates of the critical points for the graph of \( f(x) = (x - 4)(x - 2) \)?

\( f' = 2x - 6 = 0 \)

(A) 2 and 4  (B) 0, 2, and 4  (C) 2, 3, and 4  (D) 0 and 3  (E) 3

4. If \( f \) is continuous on the closed interval \([1, 3]\) with \( f''(x) < 0 \) on the open interval \((1, 3)\), then

(A) \( f(x) \) does not have a minimum on \([1, 3]\)
(B) \( f(x) \) does not have a maximum on \([1, 3]\)
(C) \( f(1) \) is the maximum value of \( f(x) \) on \([1, 3]\)
(D) \( f(1) \) is the minimum value of \( f(x) \) on \([1, 3]\)
(E) \( f(3) \) is the maximum value of \( f(x) \) on \([1, 3]\)

5. If \( f \) and \( f' \) are continuous for all \( x \), and if \( f \) has a local maximum at \( x = 4 \), then which of the following must be true?

I. \( f(4) > f(5) \)
II. \( f''(4) = 0 \)
III. \( f''(4) < 0 \)

(A) I only  (B) II only  (C) I and II only
(D) II and III only  (E) I, II, and III
6. If \( f', f'', \) and \( f''' \) are continuous for all \( x \) with \( f(5) = 9 \), and if \( f''(x) \) has a local maximum at \( x = 5 \), then
   (A) \( f'(5) = 0 \)
   (B) the graph of \( f \) is concave up in an open interval containing \( x = 5 \)
   (C) \( f \) is increasing in an open interval containing \( x = 5 \)
   (D) the point \((5, 9)\) is a point of inflection
   (E) \( f''(5) < 0 \)

7. If \( f(x) = x^2 + 2x - 8 \), then \( f(x) \) has a local minimum at
   (A) \( x = 2 \) only
   (B) \( x = 2 \) and \( x = -4 \)
   (C) \( x = -4 \) only
   (D) \( x = -1 \) only
   (E) \( x = -1 \) and \( x = 2 \)

8. If \( f(x) = xe^{-x} \), then the critical points of \( f \) are
   (A) \( x = -1 \) only
   (B) \( x = 0 \) only
   (C) \( x = 1 \) only
   (D) \( x = 0 \) and \( x = -1 \)
   (E) \( x = 0 \) and \( x = 1 \)

9. The foot of a 20 ft ladder is being pulled away from a wall at the rate of 1.5 ft/sec. At the instant when the foot is 12 ft away from the wall, the angle \( \theta \) the ladder makes with the floor is decreasing at the rate (in radians/sec) of:
   (A) \( \frac{3}{50} \)
   (B) \( \frac{1}{16} \)
   (C) \( \frac{3}{40} \)
   (D) \( \frac{1}{8} \)
   (E) \( \frac{3}{32} \)

\[
\begin{align*}
f'(x) &= x e^{-x} + e^{-x} = 0 \\
e^{-x} (1 - x) &= 0 \\
e^{-x} \neq 0 \quad 1 - x = 0 \\
x &= 1
\end{align*}
\]
10. If \( f \) and \( g \) are differentiable for all \( x \), \( h(x) = f(x) - g(x) \), and \( h(x) \) has a local maximum at \( x = 3 \), then

(A) \( f(x) \) has a relative maximum value at \( x = 3 \)

(B) \( g(x) \) has a relative minimum value at \( x = 3 \)

(C) \( f'(3) > g'(3) \)

(D) \( f'(3) = g'(3) \)

(E) \( f'(3) < g'(3) \)

\[ h'(x) = f'(x) - g'(x) \]

Then \( h'(3) = 0 \)

\[ f'(3) - g'(3) = 0 \]

\[ f'(3) = g'(3) \]

11. If \( 3p + 2q = 600 \), the maximum value of \( p \cdot q \) is

(A) 100

(B) 150

(C) 600

(D) 15,000

(E) 60,000

\[ p = \frac{200 - \frac{2}{3}q}{2} \]

Max \( p \cdot q \)

Let \( A = p \cdot q \).

\[ A = \frac{200}{3} \cdot \frac{2}{3}q \]

\[ A = 200 - \frac{4}{3}q \]

\[ q = 150 \]

\[ A = \]

\[ A' = 200 - \frac{4}{3}q = 0 \]

12. The table above gives some information about a function \( F \), for which \( F' \) and \( F'' \) are continuous for all \( x \). Both \( F' \) and \( F'' \) have exactly one zero on the interval \([2, 8]\). Which of the following statement(s) must be true?

(A) \( F(x) \) is increasing on the interval \([2, 8]\).

(B) The point \((6, 7)\) is a point of inflection.

(C) \( F(x) \) has a local minimum when \( x = 4 \).

(D) The line \( y = 7 \) is a horizontal asymptote of \( F(x) \).

(E) The line \( x = 5 \) could be a vertical asymptote of \( F(x) \).
13. The table above gives some information about a function $F$, for which $F$, $F'$, and $F''$ are continuous for all $x$. $F'$ has more than one zero, but $F''$ has exactly one zero on the interval $[2, 8]$. Which of the following statements must be false?

(A) The graph of $F(x)$ is concave down on the interval $(2, 4)$.

(B) The point $(6, 7)$ is a point of inflection.

(C) $F(x)$ has a local maximum when $x = 4$.

(D) The line $y = 7$ is not a horizontal asymptote of $F(x)$.

(E) The line $x = 5$ could be a vertical asymptote of $F(x)$.

14. The table above gives some information about a function $F$, for which $F$, $F'$, and $F''$ are continuous for all $x$. $F'$ has more than one zero, but $F''$ has exactly one zero on the interval $[2, 8]$. Which of the following statements must be true?

I. $F(x)$ has a maximum value on the interval $[2, 8]$.

II. $F(x)$ has a point of inflection for some $x$ in the interval $(2, 8)$.

III. $F(x)$ is increasing on the interval $[2, 8]$.

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

15. Let $f(x) = \frac{\sin x}{e^x}$ for $x > 0$. When the minimum value of $f(x)$ occurs, then

(A) $\sin x = 0$  
(B) $\cos x = 0$  
(C) $\cos x = \sin x$  
(D) $\cos x = -\sin x$  
(E) $f(x)$ does not have any extreme values on the interval $[0, \infty)$.

Set $f'(x) = 0$  
$(\cos x)e^x - (\sin x)e^x = 0$  
$(\cos x)e^x = (\sin x)e^x$  
$e^x(\cos x - \sin x) = 0$  
$\cos x = 0$  
$\cos x = \sin x$  
$Ignore denominator$.
16. If \( f'(x) = (x-1)^2(x+4) \), then the function \( y = f(x) \) has

- (A) one local minimum and no local maximum
- (B) no local minimum and one local maximum
- (C) one local minimum and one local maximum
- (D) one local minimum and two local maxima
- (E) two local minima and one local maximum

\[
\begin{align*}
\text{Set} & \quad f'(x) = 0 \\
(x-1)^2(x+4) & = 0 \\
x & = 1 \quad x = -4
\end{align*}
\]

17. If \( f''(x) = (x-1)^2(x-3) \cos x \), then on the interval \((0, \pi)\), how many points of inflection does the graph of \( y = f(x) \) have?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

\[
\begin{align*}
\text{Set} & \quad f''(x) = 0 \\
(x-1)^2(x-3) \cos x & = 0 \\
x & = 1 \quad x = 3 \quad \cos x = 0
\end{align*}
\]

18. If \( f(x) = \frac{\ln x}{x} \), then

- (A) \( f(x) \) has a local maximum when \( x = 1 \)
- (B) \( f(x) \) has a local minimum when \( x = 1 \)
- (C) \( f(x) \) has a local maximum when \( x = e \)
- (D) \( f(x) \) has a local minimum when \( x = e \)
- (E) \( f(x) \) has no local extreme values

\[
\begin{align*}
\frac{1}{x} - \frac{\ln x}{x} & = 0 \\
x^2 & = 1
\end{align*}
\]

19. The graph of \( y = x + \frac{1}{x} \) is both increasing and concave down on the interval

- (A) \((-\infty, -1)\)
- (B) \((-1, 0)\)
- (C) \((0, 1)\)
- (D) \((1, \infty)\)
- (E) never

\[
\begin{align*}
y' = 1 - \frac{1}{x^2} & = 0 \\
x & = \pm 1
\end{align*}
\]

\[
\begin{align*}
y'' & = 2x^{-3} = 0 \\
2 & = 0
\end{align*}
\]
20. If \( f'(x) = (x - 1)^2(x + 2) \), then \( f \) is increasing and concave down on the interval

(A) \((-\infty, -2)\)  \hspace{1cm} (B) \((-2, -1)\)  \hspace{1cm} (C) \((-2, 1)\)  \hspace{1cm} (D) \((-1, 1)\)  \hspace{1cm} (E) \((1, \infty)\)

\[
f'' = 2(x - 1)(x + 2) + (x - 1)(1) = 0
\]

\[
(x - 1)[2(x + 2) + (x - 1)] = 0
\]

\[
x = 1 \quad x = -1
\]

Think like a wiggle graph.

21. The graph of \( y = f'(x) \), the derivative of \( f \), is shown above. The function \( y = f(x) \) has a local minimum on the interval \((0, 25)\) when \( x \) equals

(A) 10 only \hspace{1cm} (B) 15 only \hspace{1cm} (C) 20 only

(D) 4 and 15 only \hspace{1cm} (E) 10 and 20

22. The graph of \( y = f'(x) \), the derivative of \( f \), is shown above. The function \( y = f(x) \) is concave down on the interval(s)

(A) \((0, 4)\) \hspace{1cm} (B) \((4, 15)\) \hspace{1cm} (C) \((10, 20)\)

(D) \((15, 25)\) \hspace{1cm} (E) \((0, 10)\) and \((20, 25)\)
23. If \( f(x) \) and \( g(x) \) are both increasing differentiable functions defined for all \( x \), then which of the following must be true?

\[ \begin{align*}
I. & \quad f(x) + g(x) \text{ is increasing} \\
II. & \quad f(x) \cdot g(x) \text{ is increasing} \\
III. & \quad f(g(x)) \text{ is increasing}
\end{align*} \]

(A) I only  (B) II only  (C) I and II only  (D) I and III only  (E) I, II, and III

24. If \( y = f(x) \) is an increasing differentiable function whose graph lies in the first quadrant and is concave down, then \( g(x) = \frac{1}{f(x)} \) is a function that is

(A) increasing and concave down  (B) decreasing and concave down  (C) increasing and concave up  (D) decreasing and concave up  (E) not enough information to determine

25. The graph of \( y = f'(x) \), the derivative of \( f \), is shown above. Which of the following is true?

\[ \begin{align*}
I. & \quad y = f(x) \text{ has a point of inflection when } x = 2 \\
II. & \quad \text{The maximum value of } f(x) \text{ occurs when } x = 2 \\
III. & \quad \text{The maximum value of } f(x) \text{ occurs when } x = 4
\end{align*} \]

(A) I only  (B) II only  (C) III only  (D) I and II only  (E) I and III only

26. The functions \( f \) and \( g \) and their tangent lines at \( (3,0) \) are shown above.

\[ \lim_{x \to 3} \frac{f(x)}{g(x)} = \frac{f(3)}{g(3)} = \frac{0}{0} \rightarrow \frac{3}{-1} \]

(A) -6  (B) -3  (C) \(-\frac{1}{3}\)  (D) \(\frac{1}{3}\)  (E) 3
27. The rate of change of the volume, \( V \), of water in a tank with respect to time, \( t \), is inversely proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) \( V(t) = k\sqrt{t} \)  
(B) \( V(t) = k\sqrt{V} \)  
(C) \( \frac{dV}{dt} = k\sqrt{t} \)  
(D) \( \frac{dV}{dt} = \frac{k}{\sqrt{V}} \)  
(E) \( \frac{dV}{dt} = k\sqrt{V} \)

28. The derivative \( g' \) of a function \( g \) is continuous and has exactly two zeros. Selected values of \( g' \) are given in the table above. If the domain of \( g \) is the set of all real numbers, then \( g \) is increasing on which of the following intervals?

(A) \(-2 \leq x \leq 2\) only  
(B) \(-1 \leq x \leq 1\) only  
(D) \(x \geq 2\) only  
(E) \(x \leq -2\) or \(x \geq 2\)

29. Let \( f''(x) = x^2 - \frac{2}{x} \). On which of the following intervals is \( f \) increasing?

(A) \((-\infty, -1]\) only  
(B) \((-\infty, 0)\) or \([\sqrt{2}, \infty)\)  
(D) \((-\infty, \sqrt{2}]\)  
(E) \([\sqrt{2}, \infty)\) only

30. The second derivative of the function \( f \) is given by \( f''(x) = -x(x-a)(x-b)^2 \). The graph of \( f'' \) is shown above. For what values of \( x \) does the graph of \( f \) have a point of inflection?

(A) \(k\) and \(b\) only  
(B) \(0\) and \(m\) only  
(C) \(a, k, j\)  
(D) \(0, m, j\)  
(E) \(a\) and \(k\) only

[Just for fun, what would the answer be if the graph was \( f'(x) = -x(x-a)(x-b)^2 \)?]
31. A particle moves along the x-axis so that at time $t \geq 0$, its position is given by $x(t) = \frac{4}{3}t^3 - 14t^2 + 49t - 53$. At what time $t$ is the particle at rest?

(A) $t = 1$ only
(B) $t = 3$ only
(C) $t = \frac{7}{2}$ only
(D) $t = 3$ and $t = \frac{7}{2}$
(E) $t = 3$ and $t = 4$

\[ v = x' = 4t^2 - 28t + 49 = 0 \]
\[ (2t - 7)(2t - 7) = 0 \]
\[ t = \frac{7}{2} \]

32. The table above shows selected values of $f(x)$ and $f'(x)$. If $f$ and $f^{-1}$ (the inverse of $f$) exist, are continuous and differentiable for $x > 0$, then $\frac{d}{dx}(f^{-1}(x))$ at $x = 1$ is $\frac{d}{dx}(f(x))\bigg|_{y=1} = -2$.

(A) -4 (B) -2 (C) $\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 2

![Stop! You may use your graphing calculator for the remainder of the test.]

**Part B**

33. A particle moves along the x-axis so that at time $t \geq 0$, its velocity is given by $v(t) = 5 - 4.9sec^2(5t)$. What is the acceleration of the particle at time $t = 3$?

(A) -32.016
(B) -0.677
(C) 19.053
(D) 44.185
(E) 72.682

\[ a(3) = v'(3) = math8 \]
34. The radius of a circle is increasing at a constant rate of 0.25 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 10π meters?

- (A) 0.25π m²/sec
- (B) 2.5π m²/sec
- (C) 5π m²/sec
- (D) 10π m²/sec
- (E) 12.5π m²/sec

Hint: \( A = \pi r^2 \quad C = 2\pi r \)

\[ \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \]

\[ \int \frac{dA}{dt} = 10\pi \frac{d}{dt} (2.5) \]

35. A particle moves along a straight path with the velocity \( v(t) = t^2 \cos t \). How many relative extrema does the particle experience on the interval \( 0 \leq t \leq 5 \)?

- (A) One
- (B) Two
- (C) Three
- (D) Four
- (E) Five

Find where the graph crosses the \( x \)-axis.

36. A particle moves along a straight path with the velocity \( v(t) = t^2 \cos t \). When will the particle be the farthest to the left on the interval \( 0 \leq t \leq 5 \)?

- (A) \( t = 0 \)
- (B) \( t = 1.570 \)
- (C) \( t = 3.644 \)
- (D) \( t = 4.712 \)
- (E) \( t = 5 \)

Find where \( v(t) \) goes from neg to pos.

37. The function \( f \) has the first derivative given by \( f'(x) = \frac{x}{x^2 + 1} \). What is the x-coordinate of the inflection point of the graph of \( f' \)?

- (A) -1 only
- (B) 0
- (C) 1 only
- (D) -1 and 1
- (E) The graph of \( f \) has no inflection point.

Find max/min on the graph of \( f'(x) \), or find where \( f'' \) crosses the \( x \)-axis.

38. Let \( f(x) = 3x^4 \) and \( g(x) = e^{3x-4} \). At what value of \( x \) does \( f \) and \( g \) have the same rate of change?

- (A) 0.127
- (B) 0.204
- (C) 0.455
- (D) 0.649
- (E) There are no such values.

\[ 12x^3 = 3e^{3x-4} \]

So/re with calculator.

39. If \( f''(x) = 3x^4 - 4x^3 - 6 \), how many critical values does \( f \) have?

- (A) One
- (B) Two
- (C) Three
- (D) Four
- (E) Five

Find when \( f' \) crosses the \( x \)-axis.
40. Which of the following is an equation of the line tangent to the graph of \( f(x) = 2x^4 + 4x^2 - 5 \) at the point where \( f'(x) = 1 \)?

(A) \( y = x - 5.062 \)  
(B) \( y = x - 3.150 \)  
(C) \( y = x \)  
(D) \( y = x - 5.685 \)  
(E) \( y = x - 1.920 \)

Use calculator to find \( f' = 1 \)
\[ 8x^3 + 8x = 1 \]
\[ x = 0.1231 \]

41. A ladder 13 feet long is leaning against a wall. If the foot of the ladder is pulled away from the wall at the rate of 0.5 feet per second, how fast will the top of the ladder be dropping at the instant when the base is 5 feet from the wall?

(A) \( -\frac{1}{12} \) ft/sec  
(B) \( -\frac{1}{8} \) ft/sec  
(C) \( -\frac{1}{6} \) ft/sec  
(D) \( -\frac{5}{24} \) ft/sec  
(E) \( -\frac{1}{4} \) ft/sec

42. Use the same situation in problem #41. Consider the area of the triangle created by the ladder and the wall. How fast is this area changing at the instant when the base is 5 feet from the wall?

(A) \( \frac{49}{4} \) ft\(^2\)/sec  
(B) \( \frac{27}{6} \) ft\(^2\)/sec  
(C) \( \frac{152}{7} \) ft\(^2\)/sec  
(D) \( \frac{193}{64} \) ft\(^2\)/sec  
(E) \( \frac{119}{48} \) ft\(^2\)/sec
43. The graph of $f'$, the derivative of the function $f$, is shown above. Which of the following statements is FALSE about $f$?

(A) $f$ is concave down for $-1 \leq x \leq 1$
(B) $f$ is decreasing for $0 \leq x \leq 2$
(C) $f$ is increasing for $-2 \leq x \leq 0$
(D) $f$ has a maximum at $x = 0$
(E) $f$ has a point of inflection at $x = -1$ and $x = 1$

**False since POI when $f'$ has a max or min. So $x = 1$ is the only POI.**