1. For what value(s) of \( x \) is \( f(x) = \frac{x}{x^2 - 1} \) discontinuous?

\[ x^2 - 1 = 0 \]
\[ x = \pm 1 \]

2. \( f(x) = \frac{x}{x^2 + 2x} \) is continuous for all real numbers EXCEPT:

\[ x^2 + 2x = 0 \]
\[ x(x + 2) = 0 \]
\[ x = 0, x = -2 \]

3. \( f(x) = \frac{x + 2}{x^2 - 4} \) is undefined at

\[ x^2 - 4 = 0 \]
\[ x = \pm 2 \]

4. \( f(x) = \frac{1}{x^2 + 1} \) is defined for all real numbers EXCEPT:

\[ x^2 + 1 = 0 \]
\[ x = 0 \]
\[ x \neq 0 \]

For #5 to #8, find the vertical asymptotes of each function.

5. \( f(x) = \frac{x}{x^2 - 49} \)

\[ x^2 - 49 = 0 \]
\[ x = 7, x = -7 \]

6. \( f(x) = \frac{x}{4x + 8} \)

\[ 4x + 8 = 0 \]
\[ x = -2 \]

7. \( f(x) = \frac{x^2 - x}{x - 1} \)

No V.A.

8. \( f(x) = \frac{(x^3 + 2x^2 - x - 2)}{x^3 + x^2 - 2x} \)

\[ \frac{x(x + 2) - 1(x + 2)}{x(x + 2) - 1} \]
\[ \frac{x(x + 2)(x - 1)}{x(x + 1)(x - 1)} \]
\[ x = 0 \]
For #9 to #12, find the horizontal asymptotes of each function.

9. \( f(x) = \frac{x^2 - 9}{3x + 2} \)

   **NO H.A.**

10. \( f(x) = \frac{x^2 - 4x + 4}{4x^2 - 1} \)

   \[ y = \frac{1}{4} \]

11. \( f(x) = \frac{x}{x^3 - 2} \)

   \[ y = 0 \]

12. \( f(x) = \frac{x^3 - 1}{x - 1} \)

   \[ \frac{(x-1)(x^2 + x + 1)}{(x-1)} \]

   **NO H.A.**

**Multiple Choice.**

13. \( f(x) = \frac{(x-1)^2}{x^2 - 1} \)

   \( \frac{(x-1)(x-1)}{(x+1)(x-1)} \)

   **VA \( x = -1 \) Hole \( x = 1 \)**

   (A) a hole at \( x = -1 \)
   (B) holes at \( x = -1 \) and \( x = 1 \)
   (C) vertical asymptotes at \( x = 1 \) and \( x = -1 \)
   (D) horizontal asymptote at \( y = -1 \)
   (E) a hole at \( x = 1 \) and a vertical asymptote at \( x = -1 \)

14. For \( f(x) = \frac{x^3 + 8}{(x+2)^2} \), choose all that are true.

   \( \frac{(x+2)(x^2 - 2x + 4)}{(x+2)(x+2)} \)

   \( x = -2 \) Hole \( x = -2 \) VA

   (A) hole at \( x = -2 \)
   (B) vertical asymptote at \( x = -2 \)
   (C) horizontal asymptote at \( y = 0 \)
   (D) horizontal asymptote at \( y = 1 \)
   (E) no horizontal asymptote
Find each characteristic of the rational functions and then sketch their graphs.

15. \( y = \frac{5x}{x - 2} \)

a) zeros
\[
\begin{align*}
x &= 0 \\
x &= 0
\end{align*}
\]

b) \( y \)-intercepts
\[
\begin{align*}
y &= s(0) \\
y &= 0
\end{align*}
\]

c) undefined values of \( x \)
\[
\begin{align*}
x - 2 &= 0 \\
x &= 2
\end{align*}
\]

\[
\text{same} \quad x = 2
\]

d) vertical asymptotes
\[
\text{essential at } x = 2
\]

jump

e) horizontal or slant asymptotes
\[
y = 5
\]

f) identify and locate all discontinuities

\[
\text{graph}
\]
16. \( y = \frac{2x}{x^2 - 1} \)

\[ \frac{-2x}{x^2 - 1} = -\left( \frac{2x}{x^2 - 1} \right) \]

a) zeros

\[ 0 = \frac{2x}{x^2 - 1} \]

\[ 0 = 2x \]

\[ 0 = x \]

b) \( y \)-intercepts

\[ y = \frac{2(0)}{0^2 - 1} \]

\[ y = 0 \]

c) undefined values of \( x \)

\[ x^2 - 1 = 0 \]

\[ x = \pm 1 \]

\[ \text{Same} \]

d) vertical asymptotes

\[ x = \pm 1 \]

e) horizontal or slant asymptotes

\[ y = 0 \]

f) identify and locate all discontinuities

\[ \text{essential at } x = \pm 1 \]
17. \( y = \frac{x^2}{x-3} \)

\[
\frac{(x^2)}{(x-3)} \rightarrow \frac{x^2}{-x-2} \quad \text{neither}
\]

a) zeros
\[
x = 0
\]

b) \( y \)-intercepts
\[
y = 0
\]

c) undefined values of \( x \)
\[
x = 3
\]

d) vertical asymptotes
\[
\]

e) horizontal or slant asymptotes
\[
\text{No HA.}
\]

f) identify and locate all discontinuities
\[
\text{Essential at } x = 3
\]

\[
\]

\[
\]

\[
\]

\[
\]

\[
\]

\[
\text{as } x \to \infty, \ y = x+3 + \frac{9}{x-3} \rightarrow y = x+3
\]
18. \( y = \frac{x^2 - 1}{x - 1} \)

a) zeros
\[
\frac{x+1}{x-1} (x-1) = 0
\]
\[
\frac{x+1}{x-1} = 0
\]
\[
x = -1
\]

b) y-intercepts
\[
y = \frac{0^2 - 1}{0 - 1}
\]
\[
y = 1
\]

c) undefined values of \( x \)
\[
(x-1) = 0
\]
\[
x = 1
\]

d) vertical asymptotes
None

e) horizontal or slant asymptotes
No HA
No SA

Since \( \frac{x^2 - 1}{x - 1} \to x + 1 \)

f) identify and locate all discontinuities
Removable at \( x = 1 \)

Graph
\[
y = x + 1
\]
with a hole at \( x = 1 \)