AP Calculus AB-4 / BC-4

The graph of the function \( f \) shown above consists of two line segments. Let \( g \) be the function given by \( g(x) = \int_{0}^{x} f(t)\,dt \).

(a) Find \( g(-1) \), \( g'(-1) \), and \( g''(-1) \).

(b) For what values of \( x \) in the open interval \((-2, 2)\) is \( g \) increasing? Explain your reasoning.

(c) For what values of \( x \) in the open interval \((-2, 2)\) is the graph of \( g \) concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of \( g \) on the closed interval \([-2, 2]\).

\[
\begin{align*}
\text{(a) } & g(-1) = \int_{0}^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -\frac{3}{2} \\
g'(-1) = f(-1) = 0 \\
g''(-1) = f'(-1) = 3
\end{align*}
\]

(b) \( g \) is increasing on \(-1 < x < 1 \) because \( g'(x) = f(x) > 0 \) on this interval.

(c) The graph of \( g \) is concave down on \( 0 < x < 2 \) because \( g''(x) = f'(x) < 0 \) on this interval.

or

because \( g'(x) = f(x) \) is decreasing on this interval.

(d) 

\[
\begin{align*}
\text{Graph of } f
\end{align*}
\]

\[
\begin{align*}
\text{(a) } & g(-2) = g(0) = g(2) = 0 \\
1: & \text{appropriate increasing/decreasing} \\
1: & \text{and concavity behavior} \\
2: & < -1 > \text{ vertical asymptote}
\end{align*}
\]
Question 5

Let \( f \) be a function defined on the closed interval \([0, 7]\). The graph of \( f \), consisting of four line segments, is shown above. Let \( g \) be the function given by \( g(x) = \int_2^x f(t) \, dt \).

(a) Find \( g(3) \), \( g'(3) \), and \( g''(3) \).

(b) Find the average rate of change of \( g \) on the interval \( 0 \leq x \leq 3 \).

(c) For how many values \( c \), where \( 0 < c < 3 \), is \( g'(c) \) equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the \( x \)-coordinate of each point of inflection of the graph of \( g \) on the interval \( 0 < x < 7 \). Justify your answer.

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(a) \[ g(3) = \int_2^3 f(t) \, dt = \frac{1}{2} (4 + 2) = 3 \]
\[ g'(3) = f(3) = 2 \]
\[ g''(3) = f'(3) = \frac{0 - 4}{4 - 2} = -2 \]

(b) \[ \frac{g(3) - g(0)}{3} = \frac{1}{3} \int_0^3 f(t) \, dt \]
\[ = \frac{1}{3} \left( \frac{1}{2} (2)(4) + \frac{1}{2} (4 + 2) \right) = \frac{7}{3} \]

(c) There are two values of \( c \).

We need \( \frac{7}{3} = g'(c) = f(c) \)

The graph of \( f \) intersects the line \( y = \frac{7}{3} \) at two places between 0 and 3.

(d) \( x = 2 \) and \( x = 5 \)

because \( g' = f \) changes from increasing to decreasing at \( x = 2 \), and from decreasing to increasing at \( x = 5 \).
The graph of the function $f$ above consists of three line segments.

(a) Let $g$ be the function given by $g(x) = \int_{-4}^{x} f(t) \, dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function $g$ defined in part (a), find the $x$-coordinate of each point of inflection of the graph of $g$ on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let $h$ be the function given by $h(x) = \int_{x}^{3} f(t) \, dt$. Find all values of $x$ in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.

(d) For the function $h$ defined in part (c), find all intervals on which $h$ is decreasing. Explain your reasoning.

| (a) $g(-1) = \int_{-4}^{-1} f(t) \, dt = -\frac{1}{2} (3)(5) = -\frac{15}{2}$ |
| $g'(-1) = f(-1) = -2$ |
| $g''(-1)$ does not exist because $f$ is not differentiable at $x = -1$. |

| (c) $x = -1, 1, 3$ |

| (d) $h$ is decreasing on $[0, 2]$ |
| $h' = -f < 0$ when $f > 0$ |