

The temperature, in degrees Celsius ($^{\circ}\text{C}$), of the water in a pond is a differentiable function W of time t . The table above shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	W ($^{\circ}\text{C}$)
0	20
3	31
6	28
9	24
12	22
15	21

- (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
- (c) A student proposes the function P , given by $P(t) = 20 + 10te^{-t/3}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.
- (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

- (a) Difference quotient; e.g.

$$W'(12) \approx \frac{W(15) - W(12)}{15 - 12} = -\frac{1}{3} \text{ }^{\circ}\text{C/day or}$$

$$W'(12) \approx \frac{W(12) - W(9)}{12 - 9} = -\frac{2}{3} \text{ }^{\circ}\text{C/day or}$$

$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = -\frac{1}{2} \text{ }^{\circ}\text{C/day}$$

(b) $\frac{3}{2}(20 + 2(31) + 2(28) + 2(24) + 2(22) + 21) = 376.5$

$$\text{Average temperature} \approx \frac{1}{15}(376.5) = 25.1 \text{ }^{\circ}\text{C}$$

(c) $P'(12) = 10e^{-t/3} - \frac{10}{3}te^{-t/3} \Big|_{t=12}$
 $= -30e^{-4} = -0.549 \text{ }^{\circ}\text{C/day}$

This means that the temperature is decreasing at the rate of $0.549 \text{ }^{\circ}\text{C/day}$ when $t = 12$ days.

(d) $\frac{1}{15} \int_0^{15} (20 + 10te^{-t/3}) dt = 25.757 \text{ }^{\circ}\text{C}$

$$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer (with units)} \end{cases}$$

$$2 : \begin{cases} 1 : \text{trapezoidal method} \\ 1 : \text{answer} \end{cases}$$

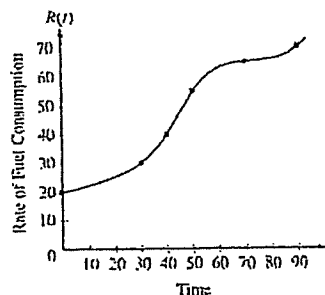
$$2 : \begin{cases} 1 : P'(12) \text{ (with or without unit)} \\ 1 : \text{interpretation} \end{cases}$$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and} \\ \quad \text{average value constant} \\ 1 : \text{answer} \end{cases}$$

Question 3

2

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

- (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
- (c) Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
- (d) For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a)
$$R'(45) \approx \frac{R(50) - R(40)}{50 - 40} = \frac{55 - 40}{10} = 1.5 \text{ gal/min}^2$$

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

(c)
$$\int_0^{90} R(t) dt \approx (30)(20) + (10)(30) + (10)(40) + (20)(55) + (20)(65) = 3700$$

Yes, this approximation is less because the graph of R is increasing on the interval.

(d) $\int_0^b R(t) dt$ is the total amount of fuel in gallons consumed for the first b minutes.
 $\frac{1}{b} \int_0^b R(t) dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.

1 : a difference quotient using numbers from table and interval that contains 45
 2 : 1 : 1.5 gal/min²

1 : $R''(45) = 0$
 2 : 1 : reason

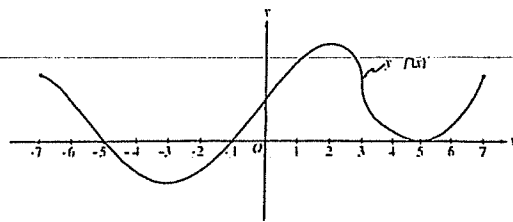
1 : value of left Riemann sum
 2 : 1 : "less" with reason

2 : meanings
 1 : meaning of $\int_0^b R(t) dt$
 3 : 1 : meaning of $\frac{1}{b} \int_0^b R(t) dt$
 < - 1 > if no reference to time b
 1 : units in both answers

3 AP Calculus AB-3

2000

The figure above shows the graph of f' , the derivative of the function f , for $-7 \leq x \leq 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.



- (a) Find all values of x , for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.
- (b) Find all values of x , for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.
- (c) Find all values of x , for $-7 < x < 7$, at which $f''(x) < 0$.
- (d) At what value of x , for $-7 \leq x \leq 7$, does f attain its absolute maximum? Justify your answer.

(a) $x = -1$

$f'(x)$ changes from negative to positive at $x = -1$

(b) $x = -5$

$f'(x)$ changes from positive to negative at $x = -5$

(c) $f''(x)$ exists and f' is decreasing on the intervals $(-7, -3)$, $(2, 3)$, and $(3, 5)$

(d) $x = 7$

The absolute maximum must occur at $x = -5$ or at an endpoint.

$f(-5) > f(-7)$ because f is increasing on $(-7, -5)$

The graph of f' shows that the magnitude of the negative change in f from $x = -5$ to $x = -1$ is smaller than the positive change in f from $x = -1$ to $x = 7$.

Therefore the net change in f is positive from $x = -5$ to $x = 7$, and $f(7) > f(-5)$. So $f(7)$ is the absolute maximum.

2 { 1: answer
1: justification

2 { 1: answer
1: justification

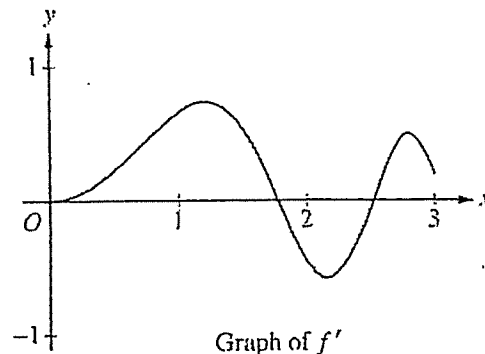
2 { 1: $(-7, -3)$
1: $(2, 3) \cup (3, 5)$

3 { 1: answer
1: identifies $x = -5$ and $x = 7$ as candidates
— or —
indicates that the graph of f increases, decreases, then increases
1: justifies $f(7) > f(-5)$

4

Question 2

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f' , the first derivative of f , given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.



- (a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.
- (b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.
- (c) Write an equation for the line tangent to the graph of f at $x = 2$.

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

(b) $f'(x) = 0$ when $x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \dots$
 On $[0, 3]$ f' changes from positive to negative only at $\sqrt{\pi}$. The absolute maximum must occur at $x = \sqrt{\pi}$ or at an endpoint.

$$f(0) = 5$$

$$f(\sqrt{\pi}) = f(0) + \int_0^{\sqrt{\pi}} f'(x) dx = 5.67911$$

$$f(3) = f(0) + \int_0^3 f'(x) dx = 5.57893$$

This shows that f has an absolute maximum at $x = \sqrt{\pi}$.

(c) $f(2) = f(0) + \int_0^2 f'(x) dx = 5.62342$

$$f'(2) = e^{-0.5} \sin(4) = -0.45902$$

$$y - 5.623 = (-0.459)(x - 2)$$

2 : { 1 : answer
1 : reason

3 : { 1 : identifies $\sqrt{\pi}$ and 3 as candidates
- or -
indicates that the graph of f increases, decreases, then increases
1 : justifies $f(\sqrt{\pi}) > f(3)$
1 : answer

4 : { 2 : $f(2)$ expression
1 : integral
1 : including $f(0)$ term
1 : $f'(2)$
1 : equation