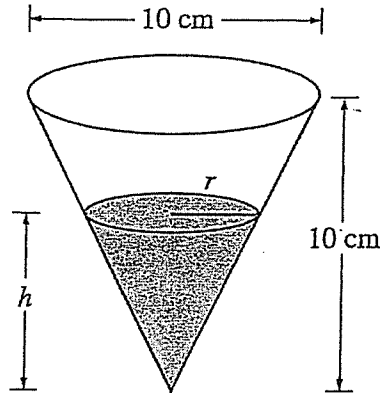


No Calculator is allowed for these problems.

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5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth  $h$  is changing at the constant rate of  $\frac{-3}{10}$  cm/hr.

(Note: The volume of a cone of height  $h$  and radius  $r$  is given by  $V = \frac{1}{3}\pi r^2 h$ .)

- Find the volume  $V$  of water in the container when  $h = 5$  cm. Indicate units of measure.
  - Find the rate of change of the volume of water in the container, with respect to time, when  $h = 5$  cm. Indicate units of measure.
  - Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?
-

5. Consider the curve given by  $y^2 = 2 + xy$ .

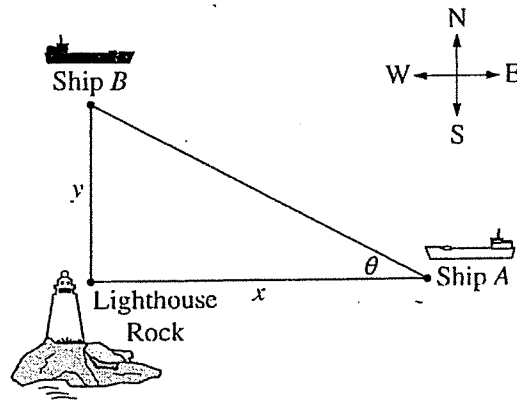
(a) Show that  $\frac{dy}{dx} = \frac{y}{2y-x}$ .

(b) Find all points  $(x, y)$  on the curve where the line tangent to the curve has slope  $\frac{1}{2}$ .

(c) Show that there are no points  $(x, y)$  on the curve where the line tangent to the curve is horizontal.

(d) Let  $x$  and  $y$  be functions of time  $t$  that are related by the equation  $y^2 = 2 + xy$ . At time  $t = 5$ , the value of  $y$  is 3 and  $\frac{dy}{dt} = 6$ . Find the value of  $\frac{dx}{dt}$  at time  $t = 5$ .

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6. Ship  $A$  is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship  $B$  is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let  $x$  be the distance between Ship  $A$  and Lighthouse Rock at time  $t$ , and let  $y$  be the distance between Ship  $B$  and Lighthouse Rock at time  $t$ , as shown in the figure above.
- Find the distance, in kilometers, between Ship  $A$  and Ship  $B$  when  $x = 4$  km and  $y = 3$  km.
  - Find the rate of change, in km/hr, of the distance between the two ships when  $x = 4$  km and  $y = 3$  km.
  - Let  $\theta$  be the angle shown in the figure. Find the rate of change of  $\theta$ , in radians per hour, when  $x = 4$  km and  $y = 3$  km.
-

6. The function  $f$  is differentiable for all real numbers. The point  $\left(3, \frac{1}{4}\right)$  is on the graph of  $y = f(x)$ , and the slope at each point  $(x, y)$  on the graph is given by  $\frac{dy}{dx} = y^2(6 - 2x)$ .

(a) Find  $\frac{d^2y}{dx^2}$  and evaluate it at the point  $\left(3, \frac{1}{4}\right)$ .

(b) Find  $y = f(x)$  by solving the differential equation  $\frac{dy}{dx} = y^2(6 - 2x)$  with the initial condition  $f(3) = \frac{1}{4}$ .

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