

## AP Calculus AB-5

A cubic polynomial function  $f$  is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where  $a$ ,  $b$ , and  $k$  are constants. The function  $f$  has a local minimum at  $x = -1$ , and the graph of  $f$  has a point of inflection at  $x = -2$ .

(a) Find the values of  $a$  and  $b$ .

(b) If  $\int_0^1 f(x) dx = 32$ , what is the value of  $k$ ?

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### Question 6

The twice-differentiable function  $f$  is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

- (a) The function  $g$  is given by  $g(x) = e^{ax} + f(x)$  for all real numbers, where  $a$  is a constant. Find  $g'(0)$  and  $g''(0)$  in terms of  $a$ . Show the work that leads to your answers.
- (b) The function  $h$  is given by  $h(x) = \cos(kx)f(x)$  for all real numbers, where  $k$  is a constant. Find  $h'(x)$  and write an equation for the line tangent to the graph of  $h$  at  $x = 0$ .
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Question 6

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Question 6!

Let  $f$  be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is  $f$  continuous at  $x = 3$ ? Explain why or why not.
- (b) Find the average value of  $f(x)$  on the closed interval  $0 \leq x \leq 5$ .
- (c) Suppose the function  $g$  is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx + 2 & \text{for } 3 < x \leq 5, \end{cases}$$

where  $k$  and  $m$  are constants. If  $g$  is differentiable at  $x = 3$ , what are the values of  $k$  and  $m$ ?

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