Chapter 2 TEST – AP Calculus Name:______________________ PT

Practice

Part A **No Calculator**

1. If $y=\dfrac{a^x}{b^x}$, when $a>0$ and $b>0$ then $\dfrac{dy}{dx}=$

(A) $\dfrac{a^x}{b^x}$ (B) $\dfrac{a^x \ln a}{b^x \ln b}$ (C) $\dfrac{a^x}{b^x}\ln\left(\dfrac{a}{b}\right)$

(D) $\dfrac{(ab)^x \ln(ab)}{b^{2x}}$ (E) $x\left(\dfrac{a}{b}\right)^{x-1}$

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2. The line tangent to the graph of $y=(x+1)e^x$ at (0 , 1) intersects the x-axis at x =

(A) -1 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) 1 (E) 2

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3. If $y=\sqrt{x^3+2x}$, then $\dfrac{dy}{dx}=$

(A) $(\frac{3}{2}x^2+1)(x^3+2x)^{-\frac{1}{2}}$ (B) $(\frac{3}{2}x^2+1)\sqrt{x^3+2x}$ (C) $(x^3+2x)^{-\frac{1}{2}}$

(D) $\frac{1}{2}(3x^2+2)^{-\frac{1}{2}}$ (E) $(3x^2+2)\sqrt{x^3+2x}$

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Part A **No Calculator**

Questions 4-5 refer to the table below. The function f is continuous and differentiable for $x > 0$ and $f(x)$ and $f'(x)$ have the indicated tabular values.

x	$f(x)$	$f'(x)$
1	2	1/2
2	3	1/3
3	1	-2

4. The equation of the line normal to $f(x)$ at $x = 2$ is:

(A) $y = -3x + 9$ (B) $y = -x + 4$ (C) $y = -\frac{1}{3}x + \frac{7}{3}$

(D) $y = -3x + 11$ (E) $y = -\frac{1}{3}x + \frac{11}{3}$

5. If f and f^{-1} (the inverse of f) exist, are continuous and differentiable for $x > 0$, then $\frac{d}{dx}\left(f^{-1}(x)\right)$ at $x = 1$ is

(A) -4 (B) -2 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 2

6. If $y = \frac{5x-4}{8x+9}$, then $\frac{dy}{dx} =$

(A) $\frac{80x-13}{(8x+9)^2}$ (B) $\frac{80x+13}{(8x+9)^2}$ (C) $\frac{77}{(8x+9)^2}$ (D) $\frac{-77}{(8x+9)^2}$ (E) $\frac{5}{8}$

Part A **No Calculator**

7. If $f(x) = \ln\left(x + \pi - e^{-0.5x}\right)$, then $f'(0)$ is

(A) $\frac{1}{2\pi}$ (B) $\frac{3}{2\pi + 1}$ (C) $\frac{1}{2(\pi - 1)}$ (D) $\frac{3}{2(\pi - 1)}$ (E) nonexistent

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8. If $y = \cos^3 x \sin 3x$, then $\frac{dy}{dx} =$

(A) $\cos^2 x\,(3\cos x \cos 3x + \sin x \sin 3x)$

(B) $\cos^2 x\,(\sin x \sin 3x - 3\cos x \cos 3x)$

(C) $3\cos^2 x\,(\cos x \cos 3x - \sin x \sin 3x)$

(D) $3\cos^2 x\,(\cos 3x - \sin 3x)$

(E) $3\cos^2 x\,(\sin 3x - \cos 3x)$

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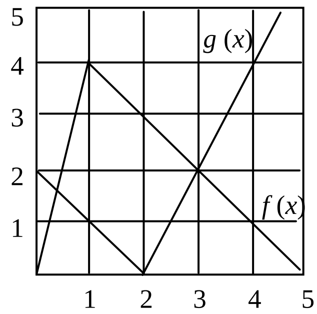
9. If $3x = \sin y$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\sqrt{9 - x^2}}$ (B) $\frac{3}{\sqrt{9 - x^2}}$ (C) $\frac{1}{\sqrt{1 - 9x^2}}$

(D) $\frac{3}{\sqrt{1 - 9x^2}}$ (E) $\frac{3}{\sqrt{9x^2 - 1}}$

Part A **No Calculator**

Questions 10-11 refer to the following graph.



10. If $h(x) = f(x) \bullet g(x)$, then $h'(3) =$

(A) -4 (B) -2 (C) 2 (D) 4 (E) 6

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11. If $m(x) = f(g(x))$, then $m'(4) =$

(A) -4 (B) -2 (C) 2 (D) 4 (E) 6

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12. If $f(x) = x^2$ and $g(x) = \sqrt{x}$ and if $h(x) = g(f(x))$, then $h'(-1) =$

(A) -2 (B) -1 (C) 0 (D) 1 (E) Does not exist

13. If $x^2 - xy + y^2 = 9$, then a vertical tangent to its curve exists at the point

(A) $(-2\sqrt{3}, -\sqrt{3})$ (B) $(\sqrt{3}, 2\sqrt{3})$ (C) $(-2\sqrt{3}, \sqrt{3})$

(D) $(-\sqrt{3}, 2\sqrt{3})$ (E) $(2\sqrt{3}, -\sqrt{3})$

14. Using local linearization for $f(x) = \sqrt{9 + \tan x}$ about $x = 0$, the approximate value of $f(0.3) =$

(A) 3 (B) 3.005 (C) 3.025 (D) 3.05 (E) 3.1

15. If $f(x) = \sin x + \cos x$, then the slope of a tangent line to $f(x)$ equals -1 at $x =$

(A) $-\frac{\pi}{2}$ (B) 0 (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{4}$

16. If $y = \ln\left(\frac{x}{y}\right)$, then at $y = 1$, $\frac{dy}{dx} =$

(A) $\frac{1-e}{e}$ (B) $\frac{1}{2e}$ (C) $\frac{2}{e}$ (D) $2e$ (E) e^2

17. If the derivative of $y = k(x)$ equals 4 when $x = -1$, what is the derivative of $y = k(1-\sqrt{x})$ when $x = 4$?

(A) -2 (B) -1 (C) 1 (D) 2 (E) 4

18. What is the slope of the line tangent to the curve $x^3 + 5x^2 y + 2y^2 = 4y + 11$ at (1 , 2) ?

(A) $-\frac{21}{5}$ (B) $-\frac{17}{6}$ (C) $-\frac{23}{9}$ (D) $-\frac{25}{12}$ (E) $-\frac{18}{13}$

Part A **No Calculator**

19. Let f be a function whose line tangent at the point (1 , 5) passes through the point (-2 , -1). Which of the following would be equal to $f'(1)$?

(A) 2 (B) 1 (C) -2 (D) 0 (E) undefined

20. If $f(x) = 5x^3 - 2x + 3$, then which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 11x - 13$ (B) $y = 11x + 13$ (C) $y = 11x + \frac{11}{3}$

(D) $y = 13x - \frac{13}{3}$ (E) $y = 13x + 13$

Stop! You may use your graphing calculator for the remainder of the test.

Part B **Graphing Calculator Required**

21. For the differential equation $\frac{dy}{dt} = \sqrt{1-2y}$, $y \neq \frac{1}{2}$, then $\frac{d^2y}{dt^2} =$

(A) -1 (B) $-\frac{1}{2}$ (C) 1 (D) $2y-1$ (E) $\frac{1}{2\sqrt{1-2y}}$

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22. If $f(x) = x^3 + x$ and $g(x)$ is the inverse of $f(x)$, then $g'(1) =$

(A) -0.5 (B) 0.003 (C) 0.077 (D) 0.25 (E) 0.417

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23. If the line $y = x + 4$ is tangent to $f(x) = ax^2 + bx$ at the point (2 , 6) , then $a + b =$

(A) 2 (B) 2.5 (C) 3 (D) 4 (E) 6

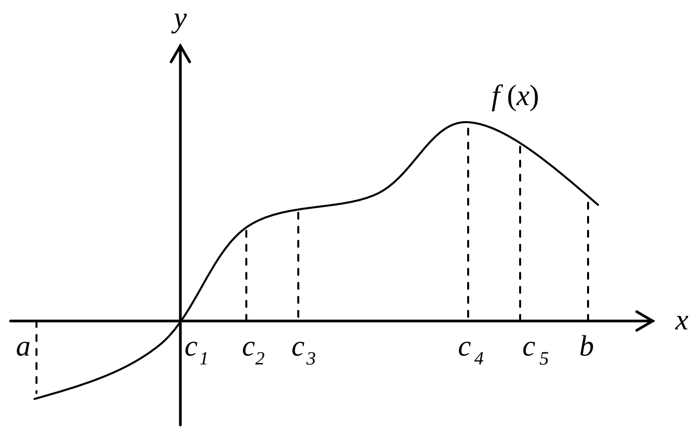
Part B **Graphing Calculator Required**

24. If $f(x) = \ln\sqrt{1+x^2}$ then a horizontal tangent line to $f(x)$ exists at $x =$

(A) $-e$ (B) -1 (C) 0 (D) $\frac{1}{e}$ (E) $\frac{1}{2}$

25. The balance, B, in a savings account t years after a deposit of \$10,000 is given by the formula $B = 10{,}000\,e^{0.075t}$. At what rate, measured in dollars per year, is the balance in the account changing at $t = 10$ years?

(A) 1000.91 (B) 1091.24 (C) 1587.75 (D) 8412.25 (E) 21,170.00



26. For the function $f(x)$ shown above, the Mean Value Theorem for Derivatives would be satisfied by which x-coordinate over the interval $a \le x \le b$?

(A) c_1 (B) c_2 (C) c_3 (D) c_4 (E) c_5

Part B **Graphing Calculator Required**

27. If f, f', and f'' are continuous on [a , b], then there is a number c in (a , b) with

(A) $f(c) = 0$

(B) $f'(c) = 0$

(C) $f''(c) = 0$

(D) the instantaneous rate of change of f at $x = c$ equal to the average rate of change of f on the interval [a , b]

(E) $f(c)$ is the maximum value of f on the interval [a , b]

28. The function $f(x)$ is continuous for the closed interval [-3 , 2] and differentiable for the open interval (-3 , 2). If $f(-3) = 1$ and $f(2) = 1$, then which of the following is true?

I. There exists c , where $-3 < c < 2$, such that $f(c) = 0$

II. There exists c , where $-3 < c < 2$, such that $f'(c) = 0$

III. There exists c , where $-3 < c < 2$, such that $f''(c) = 0$

Skip! We'll do this next chapter!

(A) I only (B) II only (C) III only

(D) I and II only (E) I, II, and III

29. The function $f(x)$ is continuous for the closed interval [-3 , 2] and differentiable for the open interval (-3 , 2). If $f(-3) = -1$ and $f(2) = 9$, then which of the following is true?

I. There exists c , where $-3 < c < 2$, such that $f(c) = 0$

II. There exists c , where $-3 < c < 2$, such that $f'(c) = 0$

III. There exists c , where $-3 < c < 2$, such that $f(c) = 2$

IV. There exists c , where $-3 < c < 2$, such that $f'(c) = 2$

Skip! We'll do this next chapter!

(A) I only (B) II only (C) III only

(D) I and III only (E) I, III, and IV

Part B **Graphing Calculator Required**

30. If $f(x) = \cos(x^3 + \pi)$, then $f'(x) =$

(A) $-3x^2 \sin(x^3 + \pi)$ (B) $-6x\sin(3x^2 + \pi)$ (C) $-(3x^2 + 1)\sin(x^3 + \pi)$

(D) $3x^2 \sin(x^3 + \pi)$ (E) $6x\sin(3x^2 + \pi)$

31. Suppose $f(2) = 3$, $f(4) = 1$, $f'(2) = -5$, and $f'(4) = 6$. Find the equation of the tangent lines of $g(x)$, $h(x)$, and $p(x)$ at $x = 2$.

(a) If $g(x) = x\,f(x)$.

(b) If $h(x) = \dfrac{f(x)}{x}$.

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(c) If $p(x) = f(x^2)$.

ANSWERS:

1 C	5 C	9 D	13 A	17 B	21 A	25 C	29 E	31a) $y = -7x + 20$
2 B	6 C	10 C	14 D	18 C	22 E	26 B	30 A	b) $y = (-13/4)x + 8$
3 A	7 D	11 B	15 D	19 A	23 D	27 D		c) $y = 24x - 47$
4 A	8 C	12 B	16 B	20 E	24 C	28 B		