Part A

1. If \( y = \frac{a^x}{b^x} \), when \( a > 0 \) and \( b > 0 \) then \( \frac{dy}{dx} = \)

   (A) \( \frac{a^x}{b^x} \)  
   (B) \( \frac{a^x \ln a}{b^x \ln b} \)  
   (C) \( \frac{a^x}{b^x \ln \left(\frac{a}{b}\right)} \)  
   (D) \( \frac{(ab)^x \ln(ab)}{b^{2x}} \)  
   (E) \( x \left(\frac{a}{b}\right)^{x-1} \)

2. The line tangent to the graph of \( y = (x+1)e^x \) at \( (0, 1) \) intersects the \( x \)-axis at \( x = \)

   (A) \(-1\)  
   (B) \(-\frac{1}{2}\)  
   (C) \(\frac{1}{2}\)  
   (D) \(1\)  
   (E) \(2\)

3. If \( y = \sqrt{x^3 + 2x} \), then \( \frac{dy}{dx} = \)

   (A) \( (\frac{3}{2}x^2 + 1)(x^3 + 2x)^{-\frac{1}{2}} \)  
   (B) \( (\frac{4}{3}x^2 + 1)\sqrt{x^3 + 2x} \)  
   (C) \( (x^3 + 2x)^{-\frac{1}{2}} \)  
   (D) \( \frac{1}{2}(3x^2 + 2)^{-\frac{1}{2}} \)  
   (E) \( (3x^2 + 2)\sqrt{x^3 + 2x} \)
Questions 4-5 refer to the table below. The function \( f \) is continuous and differentiable for \( x > 0 \) and \( f(x) \) and \( f'(x) \) have the indicated tabular values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1/3</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

4. The equation of the line normal to \( f(x) \) at \( x = 2 \) is:

(A) \( y = -3x + 9 \) \hspace{1cm} (B) \( y = -x + 4 \) \hspace{1cm} (C) \( y = -\frac{1}{3}x + \frac{7}{3} \)

(D) \( y = -3x + 11 \) \hspace{1cm} (E) \( y = -\frac{1}{3}x + \frac{11}{3} \)

5. If \( f \) and \( f^{-1} \) (the inverse of \( f \)) exist, are continuous and differentiable for \( x > 0 \), then \( \frac{d}{dx} \left( f^{-1}(x) \right) \) at \( x = 1 \) is

(A) -4 \hspace{1cm} (B) -2 \hspace{1cm} (C) \( -\frac{1}{2} \) \hspace{1cm} (D) \( \frac{1}{2} \) \hspace{1cm} (E) 2

6. If \( y = \frac{5x - 4}{8x + 9} \), then \( \frac{dy}{dx} = \)

(A) \( \frac{80x - 13}{(8x + 9)^2} \) \hspace{1cm} (B) \( \frac{80x + 13}{(8x + 9)^2} \) \hspace{1cm} (C) \( \frac{77}{(8x + 9)^2} \) \hspace{1cm} (D) \( -\frac{77}{(8x + 9)^2} \) \hspace{1cm} (E) \( \frac{5}{8} \)
7. If \( f(x) = \ln(x + \pi - e^{-0.5x}) \), then \( f'(0) \) is

<table>
<thead>
<tr>
<th></th>
<th>( \frac{1}{2\pi} )</th>
<th>( \frac{3}{2\pi + 1} )</th>
<th>( \frac{1}{2(\pi - 1)} )</th>
<th>( \frac{3}{2(\pi - 1)} )</th>
<th>nonexistent</th>
</tr>
</thead>
</table>

8. If \( y = \cos^3 x \sin 3x \), then \( \frac{dy}{dx} = \)

|   | \( \cos^2 x (3\cos x \cos 3x + \sin x \sin 3x) \) | \( \cos^2 x (\sin x \sin 3x - 3\cos x \cos 3x) \) | \( 3\cos^2 x (\cos x \cos 3x - \sin x \sin 3x) \) | \( 3\cos^2 x (3\cos x - \sin 3x) \) | \( 3\cos^2 x (\sin 3x - \cos 3x) \) |

9. If \( 3x = \sin y \), then \( \frac{dy}{dx} = \)

<table>
<thead>
<tr>
<th></th>
<th>( \frac{1}{\sqrt{9 - x^2}} )</th>
<th>( \frac{3}{\sqrt{9 - x^2}} )</th>
<th>( \frac{1}{\sqrt{1-9x^2}} )</th>
<th>( \frac{3}{\sqrt{1-9x^2}} )</th>
<th>( \frac{3}{\sqrt{9x^2 - 1}} )</th>
</tr>
</thead>
</table>

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Questions 10-11 refer to the following graph.

10. If \( h(x) = f(x) \cdot g(x) \), then \( h'(3) = \)

   (A) -4    (B) -2    (C) 2    (D) 4    (E) 6

11. If \( m(x) = f(g(x)) \), then \( m'(4) = \)

   (A) -4    (B) -2    (C) 2    (D) 4    (E) 6

12. If \( f(x) = x^2 \) and \( g(x) = \sqrt{x} \) and if \( h(x) = g(f(x)) \), then \( h'(-1) = \)

   (A) -2    (B) -1    (C) 0    (D) 1    (E) Does not exist
13. If \( x^2 - xy + y^2 = 9 \), then a vertical tangent to its curve exists at the point

14. Using local linearization for \( f(x) = \sqrt{9 + \tan x} \) about \( x = 0 \), the approximate value of \( f(0.3) = \)

15. If \( f(x) = \sin x + \cos x \), then the slope of a tangent line to \( f(x) \) equals -1 at \( x = \)

\[
\begin{array}{ccccc}
(A) & 3 & (B) & 3.005 & (C) & 3.025 \\
(D) & 3.05 & (E) & 3.1
\end{array}
\]

\[
\begin{array}{ccccc}
(A) & -\frac{\pi}{2} & (B) & 0 & (C) & \frac{\pi}{4} \\
(D) & \frac{\pi}{2} & (E) & \frac{3\pi}{4}
\end{array}
\]
16. If \( y = \ln \left( \frac{x}{y} \right) \), then at \( y = 1 \), \( \frac{dy}{dx} = \)

(\( \frac{1-e}{e} \)) (B) \( \frac{1}{2e} \) (C) \( \frac{2}{e} \) (D) \( 2e \) (E) \( e^2 \)

17. If the derivative of \( y = k(x) \) equals 4 when \( x = -1 \), what is the derivative of \( y = k(1 - \sqrt{x}) \) when \( x = 4 \) ?

(A) \(-2\) (B) \(-1\) (C) \(1\) (D) \(2\) (E) \(4\)

18. What is the slope of the line tangent to the curve \( x^3 + 5x^2y + 2y^2 = 4y + 11 \) at \((1, 2)\) ?

(A) \(-\frac{21}{5}\) (B) \(-\frac{17}{6}\) (C) \(-\frac{23}{9}\) (D) \(-\frac{25}{12}\) (E) \(-\frac{18}{13}\)
19. Let \( f \) be a function whose line tangent at the point \((1, 5)\) passes through the point \((-2, -1)\). Which of the following would be equal to \( f'(1) \)?

(A) 2  (B) 1  (C) -2  (D) 0  (E) undefined

20. If \( f(x) = 5x^3 - 2x + 3 \), then which of the following is an equation of the line tangent to the graph of \( f \) at the point where \( x = -1 \)?

(A) \( y = 11x - 13 \)  (B) \( y = 11x + 13 \)  (C) \( y = 11x + \frac{11}{3} \)

(D) \( y = 13x - \frac{13}{3} \)  (E) \( y = 13x + 13 \)

Stop! You may use your graphing calculator for the remainder of the test.
21. For the differential equation \[ \frac{dy}{dt} = \sqrt{1 - 2y}, \quad y \neq \frac{1}{2}, \] then \[ \frac{d^2 y}{dt^2} = \] 

(A) -1  
(B) -\frac{1}{2}  
(C) 1  
(D) 2y - 1  
(E) \frac{1}{2\sqrt{1 - 2y}}

22. If \( f(x) = x^3 + x \) and \( g(x) \) is the inverse of \( f(x) \), then \( g'(1) = \) 

(A) -0.5  
(B) 0.003  
(C) 0.077  
(D) 0.25  
(E) 0.417

23. If the line \( y = x + 4 \) is tangent to \( f(x) = ax^2 + bx \) at the point (2, 6), then \( a + b = \) 

(A) 2  
(B) 2.5  
(C) 3  
(D) 4  
(E) 6
24. If \( f(x) = \ln \sqrt{1 + x^2} \) then a horizontal tangent line to \( f(x) \) exists at \( x = \)

- (A) \(-e\)  
- (B) \(-1\)  
- (C) 0  
- (D) \(\frac{1}{e}\)  
- (E) \(\frac{1}{2}\)

25. The balance, \( B \), in a savings account \( t \) years after a deposit of $10,000 is given by the formula \( B = 10,000 e^{0.075t} \). At what rate, measured in dollars per year, is the balance in the account changing at \( t = 10 \) years?

- (A) 1000.91  
- (B) 1091.24  
- (C) 1587.75  
- (D) 8412.25  
- (E) 21,170.00

26. For the function \( f(x) \) shown above, the Mean Value Theorem for Derivatives would be satisfied by which \( x \)-coordinate over the interval \( a \leq x \leq b \)?

- (A) \(c_1\)  
- (B) \(c_2\)  
- (C) \(c_3\)  
- (D) \(c_4\)  
- (E) \(c_5\)
27. If \( f \), \( f' \), and \( f'' \) are continuous on \([a, b]\), then there is a number \( c \) in \((a, b)\) with

(A) \( f(c) = 0 \)
(B) \( f'(c) = 0 \)
(C) \( f''(c) = 0 \)
(D) the instantaneous rate of change of \( f \) at \( x = c \) equal to the average rate of change of \( f \) on the interval \([a, b]\)
(E) \( f(c) \) is the maximum value of \( f \) on the interval \([a, b]\)

28. The function \( f(x) \) is continuous for the closed interval \([-3, 2]\) and differentiable for the open interval \((-3, 2)\). If \( f(-3) = 1 \) and \( f(2) = 1 \), then which of the following is true?

I. There exists \( c \), where \(-3 < c < 2\), such that \( f(c) = 0 \)
II. There exists \( c \), where \(-3 < c < 2\), such that \( f'(c) = 0 \)
III. There exists \( c \), where \(-3 < c < 2\), such that \( f''(c) = 0 \)

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

29. The function \( f(x) \) is continuous for the closed interval \([-3, 2]\) and differentiable for the open interval \((-3, 2)\). If \( f(-3) = -1 \) and \( f(2) = 9 \), then which of the following is true?

I. There exists \( c \), where \(-3 < c < 2\), such that \( f(c) = 0 \)
II. There exists \( c \), where \(-3 < c < 2\), such that \( f'(c) = 0 \)
III. There exists \( c \), where \(-3 < c < 2\), such that \( f(c) = 2 \)
IV. There exists \( c \), where \(-3 < c < 2\), such that \( f'(c) = 2 \)

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, III, and IV
30. If \( f(x) = \cos(x^3 + \pi) \), then \( f'(x) = \)

(A) \(-3x^2 \sin(x^3 + \pi)\)

(B) \(-6x \sin(3x^2 + \pi)\)

(C) \(-(3x^2 + 1) \sin(x^3 + \pi)\)

(D) \(3x^2 \sin(x^3 + \pi)\)

(E) \(6x \sin(3x^2 + \pi)\)

31. Suppose \( f(2) = 3 \), \( f(4) = 1 \), \( f'(2) = -5 \), and \( f'(4) = 6 \). Find the equation of the tangent lines of \( g(x) \), \( h(x) \), and \( p(x) \) at \( x = 2 \).

(a) If \( g(x) = xf(x) \).

(b) If \( h(x) = \frac{f(x)}{x} \).
(c) If \( p(x) = f(x^2) \).