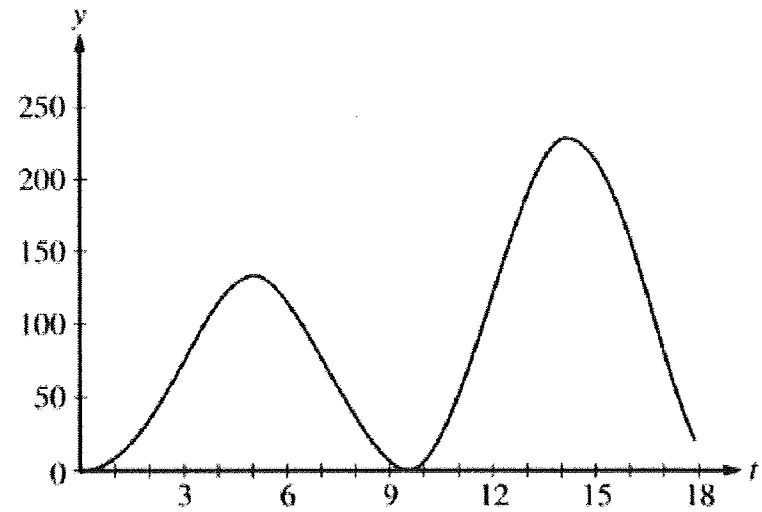
Semester 2 Unit 2 – 2- 1 WKST Name:______________________

A Graphing Calculator is required for these problems.



2. At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $L(t)$ is shown above.

   (a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

   (b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

   (c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

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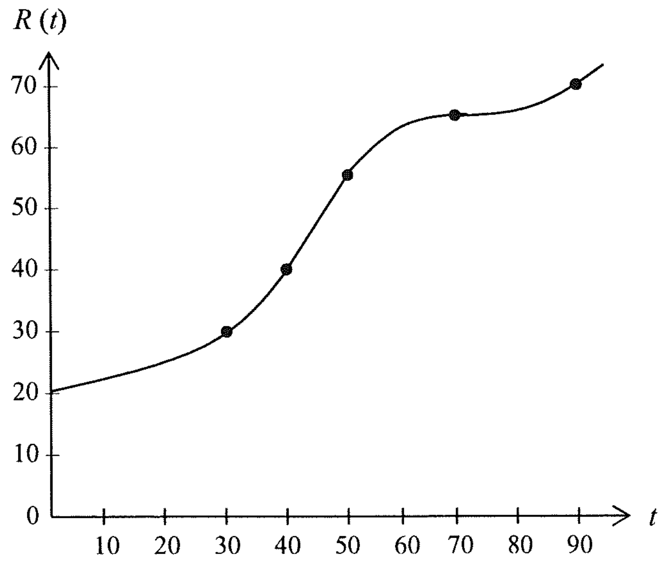
2. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use the data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is the numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning

(d) For $0 \le t \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

Sem 2 Unit 2 – 2-1 SCORING GUIDE

Question 2

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars	$2: \begin{cases} 1: \text{setup} \\ 1: \text{answer} \end{cases}$
(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$ Let $R = 12.42831$ and $S = 16.12166$ $L(t) \geq 150$ for t in the interval $[R, S]$ $\frac{1}{S-R}\int_R^S L(t)\,dt = 199.426$ cars per hour	$3: \begin{cases} 1: t\text{-interval when } L(t) \geq 150 \\ 1: \text{average value integral} \\ 1: \text{answer with units} \end{cases}$
(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400. $\int_{13}^{15} L(t)\,dt = 431.931 > 400$	$4: \begin{cases} 1: \text{considers 400 cars} \\ 1: \text{valid interval } [h, h+2] \\ 1: \text{value of } \int_h^{h+2} L(t)\,dt \\ 1: \text{answer and explanation} \end{cases}$
OR	OR
The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \geq 200$ on that interval. $L(t) \geq 200$ on any two-hour subinterval of $[13.25304, 15.32386]$. Yes, a traffic signal is required.	$4: \begin{cases} 1: \text{considers 200 cars per hour} \\ 1: \text{solves } L(t) \geq 200 \\ 1: \text{discusses 2 hour interval} \\ 1: \text{answer and explanation} \end{cases}$

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.	1 : shows that $R(6) > 0$
(b) $R'(6) = -1.913$ Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.	$2: \begin{cases} 1: \text{considers } R'(6) \\ 1: \text{answer with reason} \end{cases}$
(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$ To the nearest whole number, there are 964 mosquitoes.	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$ $R(t) > 0$ on $0 < t < 2.5\pi$ $R(t) < 0$ on $2.5\pi < t < 7.5\pi$ $R(t) > 0$ on $7.5\pi < t < 31$ The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$. $1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357,$ There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.	4 : 2 : absolute maximum value 1 : integral 1 : answer 2 : analysis 1 : computes interior critical points 1 : completes analysis

Scoring Guideline for AB Question 3

(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{55 - 40}{10}$ $= 1.5 \text{ gal/min}^2$	$2: \begin{cases} 1: \text{a difference quotient using numbers from table and interval that contains } 45 \\ 1: 1.5 \text{ gal/min}^2 \end{cases}$
(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.	$2: \begin{cases} 1: R''(45) = 0 \\ 1: \text{reason} \end{cases}$
(c) $\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + (10)(40) + (20)(55) + (20)(65) = 3700$ Yes, this approximation is less because the graph of R is increasing on the interval.	$2: \begin{cases} 1: \text{value of left Riemann sum} \\ 1: \text{“less” with reason} \end{cases}$
(d) $\int_0^b R(t)\,dt$ is the total amount of fuel in gallons consumed for the first b minutes. $\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.	$3: \begin{cases} 2: \text{meanings} \\ \quad 1: \text{meaning of } \int_0^b R(t)\,dt \\ \quad 1: \text{meaning of } \frac{1}{b}\int_0^b R(t)\,dt \\ \quad <-1> \text{ if no reference to time } b \\ 1: \text{units in both answers} \end{cases}$