Semester 2 Unit 2 – 2- 2 WKST   Name:____________________

A Graphing Calculator is required for these problems.

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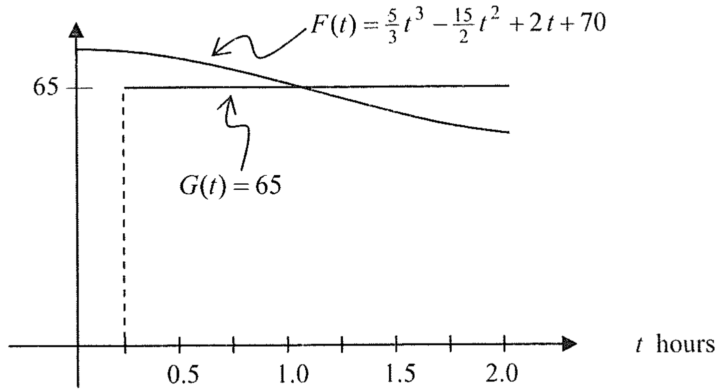
1. AP Practice Problem



The speed, measured in miles per hour, of family driving on a freeway from Jacksonville to Farmington (the location of the nearest rest stop) is modeled by the function F defined by

$$F(t)=\tfrac{5}{3}t^3-\tfrac{15}{2}t^2+2t+70.$$

The speed, measured in miles per hour, of their grandparents leaving 15 minutes later and driving on the same freeway is modeled by the function G defined by

$$G(t)=65.$$

Note that time $t=0$ for $F(t)$ represents the moment the family's car enters the freeway in Jacksonville. Similarly, time $t=0.25$ for $G(t)$ represents the moment the grandparent's car enters the freeway 15 minutes later. In both cases their speed on the onramp and offramps are ignored.

(a) How much of a head start (in miles) does the family have once the grandparents enter the freeway?

(b) Find the rate at which the total the distance between the family and the grandparents is increasing at time $t=0.5$.

(c) At what time t, for $0.25\le t\le 2.0$, does the model predict that the distance between the family and the grandparents is a maximum? Justify your answer. How much farther ahead is the family at that point?

(d) The family will arrive at Farmington before their grandparents at time $t=2.0$. About how long must the family wait until their grandparents arrive?

(e) Suppose both the family and their grandparents bypass the rest stop at Farmington completely and stay on the freeway. Write an integral equation to find at what time T the grandparents will overtake the family. Use antiderivatives to find that time.

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2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

GO ON TO THE NEXT PAGE.

2. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

Sem 2 Unit 2 – 2-2 SCORING GUIDE

ANSWERS:

1a) 17.525 miles

b) 4.333 miles per hour

c) $F-G$ changes from positive to negative at $t = 1.140$ hours.
The family is 20.249 miles ahead.

d) 15 minutes, 36.923 seconds

e) $\int_0^T \frac{5}{3}t^3 - \frac{15}{2}t^2 + 2t + 70\,dt = \int_{0.25}^T 65\,dt$

$T = 3.683$ hrs, at which the distance traveled will be 223.136 miles

AP Calculus AB–2 / BC-2 Final Draft for Scoring

(a) $\int_9^{17} E(t)\,dt = 6004.270$ 6004 people entered the park by 5 pm.	3 { 1 : limits 1 : integrand 1 : answer
(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$ The amount collected was \$104,048. or $\int_{17}^{23} E(t)\,dt = 1271.283$ 1271 people entered the park between 5 pm and 11 pm, so the amount collected was $\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$	1 : setup
(c) $H'(17) = E(17) - L(17) = -380.281$ There were 3725 people in the park at $t = 17$. The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.	3 { 1 : value of $H'(17)$ 2 : meanings 1 : meaning of $H(17)$ 1 : meaning of $H'(17)$ $<-1>$ if no reference to $t = 17$
(d) $H'(t) = E(t) - L(t) = 0$ $t = 15.794$ or 15.795	2 { 1 : $E(t) - L(t) = 0$. 1 : answer

AP® CALCULUS AB
SCORING GUIDELINES

Question 2

(a) $\int_0^6 R(t)\,dt = 31.815$ or 31.816 yd^3

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer with units} \end{cases}$

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x))\,dx$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or -1.909 yd^3/hr

1 : answer

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.
The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

$3 : \begin{cases} 1 : \text{sets } Y'(t) = 0 \\ 1 : \text{critical } t\text{-value} \\ 1 : \text{answer with justification} \end{cases}$