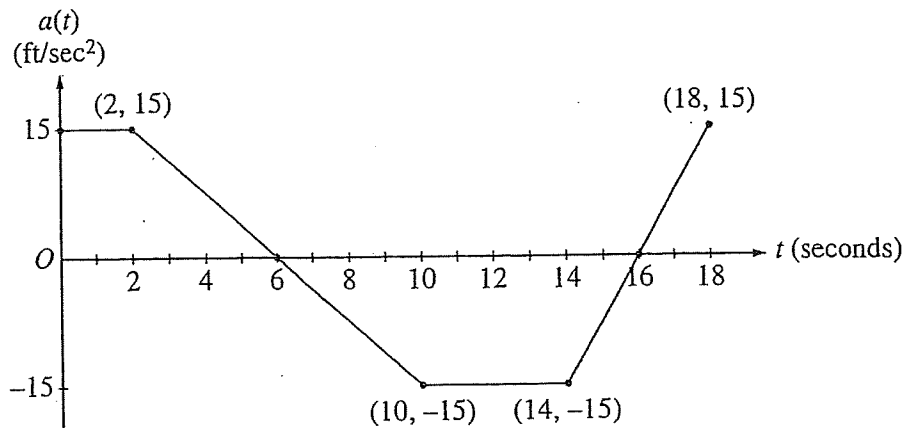


A Graphing Calculator is required for the first three problems, but is not allowed for the last two problems.



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \leq t \leq 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2 , is the piecewise linear function defined by the graph above.
- Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?
 - At what time in the interval $0 \leq t \leq 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?
 - On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
 - At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.
-

Calculator allowed.

3. An object moves along the x -axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

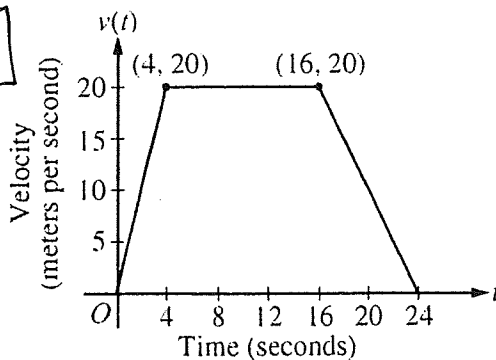
(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

Calculator allowed

3. A particle moves along the y -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)
- (a) Find the acceleration of the particle at time $t = 2$.
 - (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
 - (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.
 - (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.
-

No Calculator



5. A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.
- (a) Find $\int_0^{24} v(t) dt$. Using correct units, explain the meaning of $\int_0^{24} v(t) dt$.
- (b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
- (c) Let $a(t)$ be the car's acceleration at time t , in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
- (d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?
-

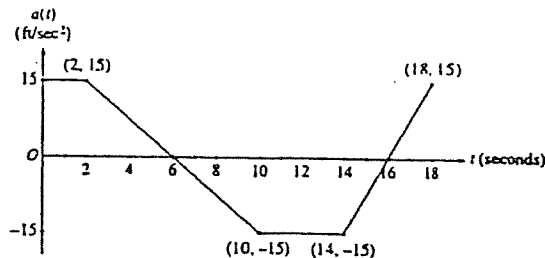
No Calculators

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.
- (a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$.
- (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

AP Calculus AB-3 / BC-3 Final Draft for Scoring

A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \leq t \leq 18$ seconds, the car's acceleration $a(t)$, in ft/sec², is the piecewise linear function defined by the graph above.



- (a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?
- (b) At what time in the interval $0 \leq t \leq 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?
- (c) On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
- (d) At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.

(a) Since $v'(2) = a(2)$ and $a(2) = 15 > 0$, the velocity is increasing at $t = 2$.

(b) At time $t = 12$ because

$$v(12) - v(0) = \int_0^{12} a(t) dt = 0.$$

(c) The absolute maximum velocity is 115 ft/sec at $t = 6$.

The absolute maximum must occur at $t = 6$ or at an endpoint.

$$\begin{aligned} v(6) &= 55 + \int_0^6 a(t) dt \\ &= 55 + 2(15) + \frac{1}{2}(4)(15) = 115 > v(0) \end{aligned}$$

$$\int_6^{18} a(t) dt < 0 \text{ so } v(18) < v(6)$$

(d) The car's velocity is never equal to 0. The absolute minimum occurs at $t = 16$ where

$$v(16) = 115 + \int_6^{16} a(t) dt = 115 - 105 = 10 > 0.$$

1 : answer and reason

2 : $\begin{cases} 1 : t = 12 \\ 1 : \text{reason} \end{cases}$

4 : $\begin{cases} 1 : t = 6 \\ 1 : \text{absolute maximum velocity} \\ 1 : \text{identifies } t = 6 \text{ and } t = 18 \text{ as candidates} \\ \text{or} \\ \text{indicates that } v \text{ increases, decreases, then increases} \\ 1 : \text{eliminates } t = 18 \end{cases}$

2 : $\begin{cases} 1 : \text{at no time } t \\ 1 : \text{reason} \end{cases}$

An object moves along the x -axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

$$\begin{aligned} \text{(a)} \quad a(4) &= v'(4) = \frac{\pi}{3} \cos\left(\frac{4\pi}{3}\right) \\ &= -\frac{\pi}{6} \text{ or } -0.523 \text{ or } -0.524 \end{aligned}$$

(b) On $3 < t < 4.5$:

$$a(t) = v'(t) = \frac{\pi}{3} \cos\left(\frac{\pi}{3}t\right) < 0$$

Statement I is correct since $a(t) < 0$.

Statement II is correct since $v(t) < 0$ and $a(t) < 0$.

$$\text{(c)} \quad \text{Distance} = \int_0^4 |v(t)| dt = 2.387$$

OR

$$x(t) = -\frac{3}{\pi} \cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$$

$$x(0) = 2$$

$$x(4) = 2 + \frac{9}{2\pi} = 3.43239$$

$$v(t) = 0 \text{ when } t = 3$$

$$x(3) = \frac{6}{\pi} + 2 = 3.90986$$

$$|x(3) - x(0)| + |x(4) - x(3)| = \frac{15}{2\pi} = 2.387$$

$$\text{(d)} \quad x(4) = x(0) + \int_0^4 v(t) dt = 3.432$$

OR

$$x(t) = -\frac{3}{\pi} \cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$$

$$x(4) = 2 + \frac{9}{2\pi} = 3.432$$

1 : answer

3 { 1 : I correct, with reason
1 : II correct
1 : reason for II

3 { 1 : { limits of 0 and 4 on an integral
of $v(t)$ or $|v(t)|$
or
uses $x(0)$ and $x(4)$ to compute
distance
1 : handles change of direction at
student's turning point
1 : answer
0/1 if incorrect turning point or
no turning point

2 { 1 : integral
1 : answer

2 { 1 : $x(t) = -\frac{3}{\pi} \cos\left(\frac{\pi}{3}t\right) + C$
1 : answer
0/1 if no constant of integration

**AP[®] CALCULUS AB
SCORING GUIDELINES**

Question 3

A particle moves along the y -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$.

At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

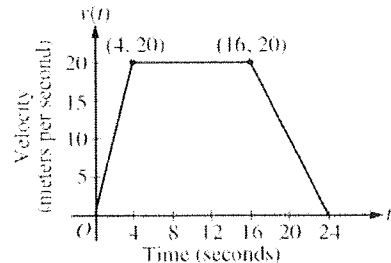
- (a) Find the acceleration of the particle at time $t = 2$.
- (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
- (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.
- (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

<p>(a) $a(2) = v'(2) = -0.132$ or -0.133</p>	<p>1 : answer</p>
<p>(b) $v(2) = -0.436$ Speed is increasing since $a(2) < 0$ and $v(2) < 0$.</p>	<p>1 : answer with reason</p>
<p>(c) $v(t) = 0$ when $\tan^{-1}(e^t) = 1$ $t = \ln(\tan(1)) = 0.443$ is the only critical value for y. $v(t) > 0$ for $0 < t < \ln(\tan(1))$ $v(t) < 0$ for $t > \ln(\tan(1))$ $y(t)$ has an absolute maximum at $t = 0.443$.</p>	<p>3 : $\left\{ \begin{array}{l} 1 : \text{sets } v(t) = 0 \\ 1 : \text{identifies } t = 0.443 \text{ as a candidate} \\ 1 : \text{justifies absolute maximum} \end{array} \right.$</p>
<p>(d) $y(2) = -1 + \int_0^2 v(t) dt = -1.360$ or -1.361 The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.</p>	<p>4 : $\left\{ \begin{array}{l} 1 : \int_0^2 v(t) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{value of } y(2) \\ 1 : \text{answer with reason} \end{array} \right.$</p>

**AP[®] CALCULUS AB
SCORING GUIDELINES**

Question 5

A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.



- (a) Find $\int_0^{24} v(t) dt$. Using correct units, explain the meaning of $\int_0^{24} v(t) dt$.
- (b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
- (c) Let $a(t)$ be the car's acceleration at time t , in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
- (d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(a) $\int_0^{24} v(t) dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$
The car travels 360 meters in these 24 seconds.

2 : $\begin{cases} 1 : \text{value} \\ 1 : \text{meaning with units} \end{cases}$

(b) $v'(4)$ does not exist because

$$\lim_{t \rightarrow 4^-} \left(\frac{v(t) - v(4)}{t - 4} \right) = 5 \neq 0 = \lim_{t \rightarrow 4^+} \left(\frac{v(t) - v(4)}{t - 4} \right).$$

$$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

3 : $\begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$

(c)
$$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$

 $a(t)$ does not exist at $t = 4$ and $t = 16$.

2 : $\begin{cases} 1 : \text{finds the values } 5, 0, -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of v on $[8, 20]$ is

$$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2.$$

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

2 : $\begin{cases} 1 : \text{average rate of change of } v \text{ on } [8, 20] \\ 1 : \text{answer with explanation} \end{cases}$

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.

- (a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$.
- (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

- (a) Average acceleration of rocket A is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2$$

- (b) Since the velocity is positive, $\int_{10}^{70} v(t) dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

A midpoint Riemann sum is

$$20[v(20) + v(40) + v(60)]$$

$$= 20[22 + 35 + 44] = 2020 \text{ ft}$$

- (c) Let $v_B(t)$ be the velocity of rocket B at time t .

$$v_B(t) = \int \frac{3}{\sqrt{t+1}} dt = 6\sqrt{t+1} + C$$

$$2 = v_B(0) = 6 + C$$

$$v_B(t) = 6\sqrt{t+1} - 4$$

$$v_B(80) = 50 > 49 = v(80)$$

Rocket B is traveling faster at time $t = 80$ seconds.

Units of ft/sec^2 in (a) and ft in (b)

1 : answer

3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{uses } v(20), v(40), v(60) \\ 1 : \text{value} \end{cases}$

4 : $\begin{cases} 1 : 6\sqrt{t+1} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{finds } v_B(80), \text{ compares to } v(80), \\ \text{and draws a conclusion} \end{cases}$

1 : units in (a) and (b)