

2-1 Obj:

to find total change, quantity, avg, etc. from a given rate.

$R(t) = -t^4 + 15t$ rate water flows into a tank (gal/min). There were 100 gal at $t=0$.

Q How much water flowed into the tank after 2 min?

Total Δ in Antideriv

$$\text{FTC } \int_a^b \text{Rate } dt = F(b) - F(a)$$

$$\int_0^2 R(t) dt = 23.6 \text{ gal}$$

Q How much in the tank after 2 min?

Re-write FTC $F(b) = F(a) + \int_a^b \text{Rate } dt$

$$\begin{aligned} F(2) &= F(0) + \int_0^2 R(t) dt \\ &= 100 + 23.6 \\ &= \boxed{123.6 \text{ gal}} \end{aligned}$$

Q What is the average rate that water flows into the tank over the first 3 minutes?

$$\text{Avg } R(t) = \frac{1}{b-a} \int_a^b R(t) dt$$

$$= \frac{1}{3-0} \int_0^3 R(t) dt$$

$$= \boxed{6.3 \text{ gal/min}}$$

Q What is the maximum amount of water over the 3 min?

ie Find Absolute Max

- Find Critical Pts $[R'(t) = 0]$

- Test Critical Pts & end pts using FTC or

finding the antiderivative.

$$R(t) = 0$$

$$= t^4 + 15t = 0$$

$$= t(t^3 + 15) = 0$$

$$t = 0 \quad t = \sqrt[3]{-15}$$

Test $t=0$, $t=\sqrt[3]{15}$, and $t=3$.

t	$F(t) = 100 + \int_0^t R(t) dt$
0	100
$\sqrt[3]{15}$	$100 + \int_0^{\sqrt[3]{15}} R(t) dt = 127.3699$
3	$100 + \int_0^3 R(t) dt = 118.9$

\therefore Abs max at $t = \sqrt[3]{15}$ min
of 127.370 gal