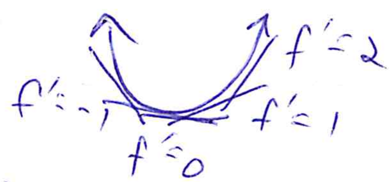
$\boxed{3.4}$ Obj : to use $f''(x)$ to find concavity, pts of inflection, & relative extrema.

$\boxed{3.5}$ Obj : to use f' & f'' to sketch $f(x)$.

$\boxed{\text{Concavity}}$ Let $f(x)$ be continuous at $x=p$.

If $f''(p)>0$, then $f'(p)$ increasing, and $f(x)$ <u>concave up</u>.



If $f''(p)<0$, then $f'(p)$ decreasing, and $f(x)$ concave down

If $f''(p)=0$, the $f'(p)$ horz tangent, and $f(x)$ has a <u>possible point of inflection</u>

If $f''(x)$ has a sign change at $x=p$, then $x=p$ is a <u>point of inflection</u>.

If $f''(x)=0$ for an interval (a,b), then $f(x)$ is linear on (a,b).

Ex: Where is $f(x) = xe^{-x}$ concave down?

$$f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x}(-1)$$

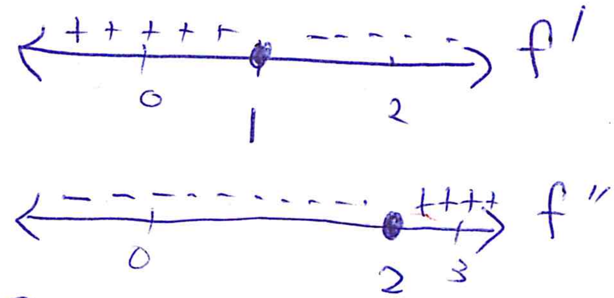
$$e^{-x} - xe^{-x}$$

$$f' = \boxed{e^{-x}(1-x)} = 0$$

$\cancel{e^{-x} = 0}$ $\qquad$ $\boxed{1-x} = 0$

$x = 1$

$\boxed{\text{rel max @ } x=1}$

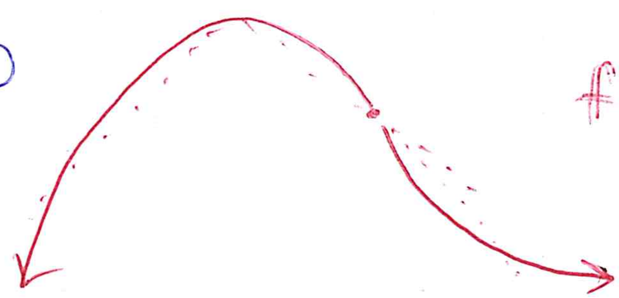


$$f'' = e^{-x}(-1)(1-x) + e^{-x}(-1) = 0$$

$$-e^{-x}(1 - x + 1) = 0$$

$\cancel{-e^{-x} = 0}$ $\qquad$ $2 - x = 0$

$x = 2$



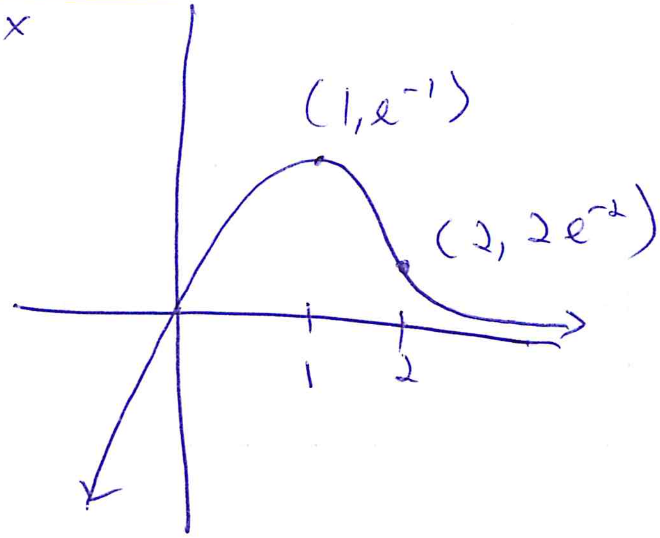
concave up $(2, +\infty)$

concave down $(-\infty, 2)$

POI @ $x = 2$

Now Graph the function.

x	$f(x)=xe^{-x}$
good → 0	0
Crit. pt → 1	e^{-1}
POIs → 2	$2e^{-2}$



$\boxed{\text{2nd Derivative Test}}$ Let $x=c$ be a critical pt.

If $f'(c)=0$ and $f''(c)>0$, then
$x=c$ is a <u>Relative Minimum</u>

If $f'(c)=0$ and $f''(c)<0$, then
$x=c$ is a <u>Relative Maximum</u>.

$\boxed{\text{Another example}}$

$f(x)=x^4-4x^3$