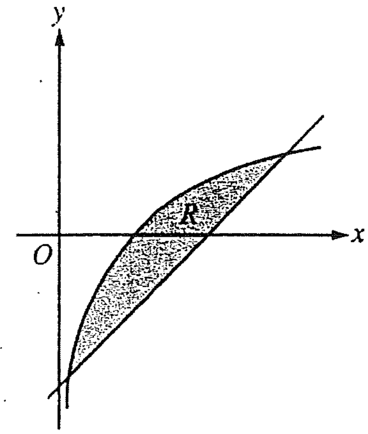
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

Unit 1 Practice Test

A graphing calculator is required for some problems or parts of problems.

2006



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

2002

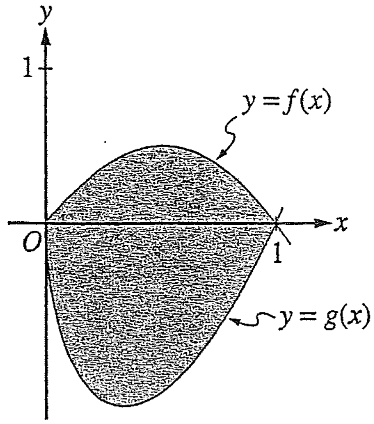
1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

   (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

   (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

   (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

GO ON TO THE NEXT PAGE.



2004

2. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

   (a) Find the area of the shaded region enclosed by the graphs of f and g.

   (b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

   (c) Suppose the shaded region models the base of a 3-dimensional figure. At all points in the shaded region at a distance x from the y-axis, the height is given by $h(x) = 6x(1-x)$. Find the volume of the 3-dimensional figure.

   (d) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

WRITE ALL WORK IN THE PINK TEST BOOKLET.

Unit 1 Practice Test Answers

Calculus AB Sample Free-Response Questions

2006

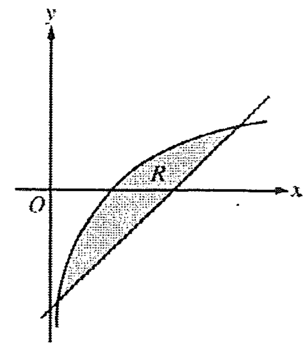
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x-2))\,dx = 1.949$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi \int_S^T \left((\ln(x)+3)^2 - (x-2+3)^2 \right) dx$ $= 34.198$ or 34.199	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits, constant, and answer} \end{cases}$
(c) Volume $= \pi \int_{S-2}^{T-2} \left((y+2)^2 - \left(e^y\right)^2 \right) dy$	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits and constant} \end{cases}$

AP Calculus AB–1 / BC–1 Final Draft for Scoring

2002

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

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(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

(a) Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ or 1.223

$$2\begin{cases} 1: \text{ integral} \\ 1: \text{ answer} \end{cases}$$

(b) Volume $= \pi \int_{1/2}^{1} \left((4 - \ln x)^2 - (4 - e^x)^2 \right) dx$

$= 7.515\pi$ or 23.609

$$4\begin{cases} 1: \text{ limits and constant} \\ 2: \text{ integrand} \\ \quad < -1 > \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if not of the form} \\ \quad k\int_a^b \left(R(x)^2 - r(x)^2 \right) dx \\ 1: \text{ answer} \end{cases}$$

(c) $h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0$

$x = 0.567143$

Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints.

$h(0.567143) = 2.330$

$h(0.5) = 2.3418$

$h(1) = 2.718$

The absolute minimum is 2.330.

The absolute maximum is 2.718.

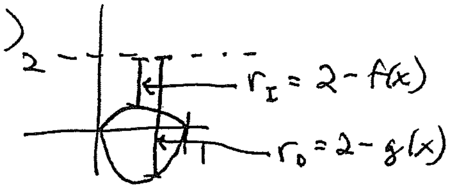
$$3\begin{cases} 1: \text{ considers } h'(x) = 0 \\ 1: \text{ identifies critical point} \\ \quad \text{and endpoints as candidates} \\ 1: \text{ answers} \end{cases}$$

Note: Errors in computation come off the third point.

$\boxed{2004}$

(2) a) $\int_0^1 f(x) - g(x)\,dx = \boxed{1.133}$

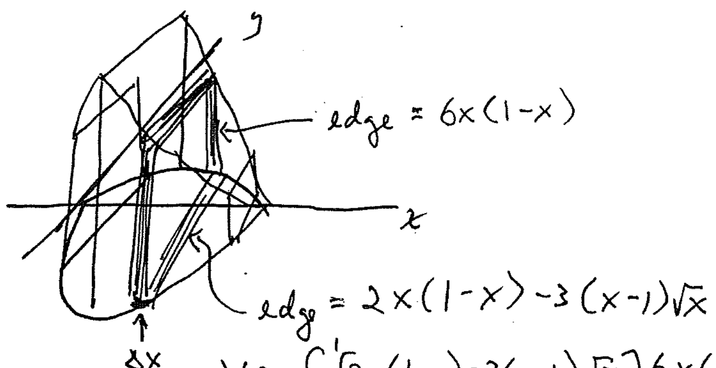
b)



$$V = \int_0^1 \pi\left(2 - g(x)\right)^2 - \pi\left(2 - f(x)\right)^2 dx$$

$$= \boxed{16.179}$$

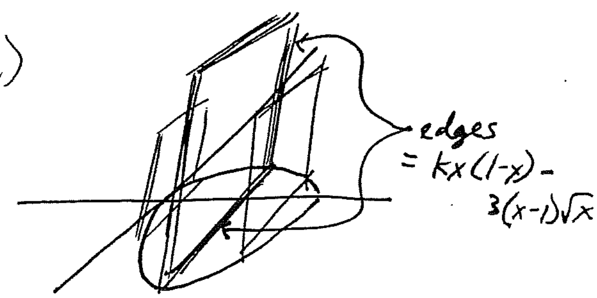
c)



$$V = \int_0^1 [2x(1-x) - 3(x-1)\sqrt{x}]\,6x(1-x)\,dx$$

$$= \boxed{1.314}$$

d)



$$V = \int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x}\right)^2 dx = 15$$

Just for fun… $k = 16.60398074$