Unit 1  Comprehensive Review  Name: KEY

Linear Functions, Exponential Functions, Domain and Range, Transformations, Odd/Even Functions, Composition Functions, Inverses, Logarithms, Trigonometry, and Polynomials

Try every problem without a calculator first. Use your calculator only when needed.

1. The table below shows the decay of certain bacteria for a 5-day period.

<table>
<thead>
<tr>
<th>Day</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm²</td>
<td>3.50</td>
<td>3.25</td>
<td>3.00</td>
<td>2.75</td>
<td>2.50</td>
<td>2.25</td>
</tr>
</tbody>
</table>

a) Write a **linear function** that models the situation.

\[ y = mx + b \]

\[ m = \frac{\Delta y}{\Delta x} \]

\[ m = \frac{-0.25}{1} \]

\[ y = -0.25x + 3.50 \]

2. The table below shows the decay of another bacteria for a 5-day period.

<table>
<thead>
<tr>
<th>Day</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm²</td>
<td>3.50</td>
<td>3.25</td>
<td>3.02</td>
<td>2.80</td>
<td>2.60</td>
<td>2.41</td>
</tr>
</tbody>
</table>

Write an **exponential function** \( y = ba^x \) that models the situation.

\[ a = \frac{y_n}{y_{n-1}} \]

\[ a = \frac{3.25}{3.5} \]

\[ y = 3.5 \left( \frac{3.25}{3.5} \right)^x \]
3. In 2002, Mr. Tactay yearly income was $30,000. In 2012, Mr. Tactay yearly income was $46,000. Assuming linear growth, what will Mr. Tactay’s income be in 2020?

\[
\begin{align*}
(2,30) & \quad (12,46) \\
\text{No b is given} & \\
(2,30) & \quad (12,46) \\
\text{\( m = \frac{46-30}{12-2} \)} & \\
\text{\( m = 1.6 \)} & \\
\text{\( y - y_1 = m(x-x_1) \)} & \\
\text{\( y - 30 = 1.6(x-2) \)} & \\
\text{\( y(20) = 1.6(20) + 30 \)} & \\
\text{\( y = 1.6x + 26.8 \)} & \\
\end{align*}
\]

4. In 2002, Mr. Tactay yearly income was $30,000. In 2012, Mr. Tactay yearly income was $46,000. Assuming exponential growth, what will Mr. Tactay’s income be in 2020?

\[
\begin{align*}
(2,30) & \quad (12,46) \\
30 = b \cdot a^2 & \quad 46 = b \cdot a^{1.2} \\
30 = b & \quad 46 = 30a^{1.2} \\
\text{\( a = \sqrt[1.2]{\frac{46}{30}} \)} & \\
\text{\( 10 \sqrt[1.2]{\frac{46}{30}} = a \)} & \\
\text{\( b = \frac{30}{(a^{1.2})^2} = \frac{30}{(41.8^{1.2})^2} \)} & \\
\text{\( y \in 27.549(1.0436^x) \)} & \\
\text{at \( x = 20 \),} & \\
\text{\( y = 27.549(1.0436^{20}) \)} & \\
\text{\( y = 641,754 \)} & \\
\end{align*}
\]
5. Write the exponential equation that is modeled by the graph.

Formula: \[ a = \sqrt{\frac{y_m}{y_n}} \]

If \( m > r \)

\[ a = \sqrt{8} = 2 \sqrt{2} \]

\[ y = b \cdot (2 \sqrt{2})^x \]

At \((3, 1)\):

\[ 1 = b \cdot (2 \sqrt{2})^3 \]

\[ b = \frac{1}{(2 \sqrt{2})^3} \]

6. The population of Calculusville in 2000 is 50,000. Calculusville has an annual growth of 3%.
   a) How long until the population is 75,000?
   b) What continuous growth rate would be equivalent to the annual growth of 3%?

\[ y = 50,000 \cdot (1.03)^x \]

75,000 = 50,000 \cdot (1.03)^x

1.5 = 1.03 +

\[ \ln 1.5 = x \ln 1.03 \]

\[ x = \frac{\ln 1.5}{\ln 1.03} \]

\[ x \approx 13.767 \text{ yrs} \]

Annual:

\[ y = P(1+r)^t \]

Continuous:

\[ y = Pe^{rt} \]

\[ 1.03 = e \]

\[ \ln 1.03 = \ln e \]

\[ \ln 1.03 = r \]

\[ r = 0.0295588 \]

\[ 2.956 \% \]
7. Write an equation to model each situation.
   a) The amount of money you earn working at McBurgerQueen varies directly with the number of hours you work. You earned $185 when you worked 20 hours. Hint: A direct variation is modeled by \( y = kx \).

\[
\begin{align*}
M &= k \cdot h \\
185 &= k \cdot 20 \\
\frac{185}{20} &= k
\end{align*}
\]

\[
y = \frac{185}{20}
\]

b) \( Q \) varies inversely with square of \( t \). \( Q(2.5) = 12 \). Hint: An inverse variation is modeled by \( y = \frac{k}{x^2} \).

\[
\begin{align*}
Q &= \frac{k}{t^2} \\
12 &= \frac{k}{(2.5)^2} \\
75 &= k
\end{align*}
\]

8. Find domain and range.
   a) \( y = \sqrt{x+2} \)
   b) \( y = \sqrt[3]{x} \)
   c) \( y = e^x \)
   d) \( y = \log x \)

\[
\begin{align*}
\text{D: } &x \geq -2 & \text{D: } &R & \text{D: } &R & \text{D: } &x > 0 \\
\text{R: } &y > 0 & \text{R: } &R & \text{R: } &y > 0 & \text{R: } &R
\end{align*}
\]

9. Find domain and range. They all appear to simplify to \( x \), but not always.
   a) \( y = \frac{x^2}{x} = x \)
   b) \( y = |x| \)
   c) \( y = \sqrt{x^2} \)
   d) \( y = (\sqrt{x})^2 \)
   e) \( y = 10^{\log x} \)
   f) \( y = \ln e^x = x \)

\[
\begin{align*}
\text{D: } &x \neq 0 & \text{D: } &R & \text{D: } &R & \text{D: } &x > 0 \\
\text{R: } &y \neq 0 & \text{R: } &y \geq 0 & \text{R: } &y > 0 & &
\end{align*}
\]
10. Use the graph on the right. State YES or NO if the functions are represented by the graphs below them.

a) \( y = 2f(x) \)  
\[ \text{NO} \]

b) \( y = f(x+1) \)  
\[ \text{NO} \]

c) \( y = 1 - f(x) \)  
\[ \text{YES} \]

11. Write an equation to represent each transformation.

a) \( y = e^x \) is shifted right 2 units and down 3 units
\[ y = e^{x-2} - 3 \]

b) \( y = \sqrt{x} \) is reflected over the x-axis.
\[ y = -\sqrt{x} \]

12. Consider \( f(x) = x-2 \) and \( g(x) = x^4 \).

a) Find \( y = f \circ g \).
\[ f(g(x)) = (x^4 - 2) \]

b) Find \( y = g \circ f \). (jff - Expand using Pascal's Triangle.)
\[ g(f(x)) = (x - 1)^2 \]
\[ 1x^4 - 4x^3 + 6x^2 - 4x + 1 \]
\[ x^4 - 8x^3 + 24x^2 - 32x + 16 \]
13. Evaluate \( f(2) \) for \( f(x) = x^4 - 3x^3 - x + 1 \) using
   a) Synthetic Division
   b) the STO function on your calculator

   \[
   \begin{array}{c|cccc}
   2 & 1 & -3 & 0 & -1 & 1 \\
   \hline
   & 2 & -2 & -4 & -10 \\
   \hline
   & 1 & -1 & 2 & -5 & -9 \\
   \end{array}
   \]

   \[f(2) = -9\]

14. Determine if the functions are odd, even, or neither. Hint: Odd if \( f(-x) = -f(x) \). Even if \( f(-x) = f(x) \).
   a) \( f(x) = \frac{1}{x^2} \)
   b) \( y = x^3 - x \)
   c) \( f(x) = x^3 + 1 \)

   Even
   \[f(-x) = (-x)^3 - (-x) = -x^3 + x\]
   \(f(x) = (x)^3 + 1\)

   Odd
   \[f(-x) = -f(x)\]

   d) \( f(x) = xe^{x^2} \)
   e) \( y = \log_3 x^3 \)

   \(f(-x) = (-x)e^{(-x)^2} = -xe^{x^2}\)
   \[f(-x) = \log_3(-x)^3 = \log(-x^3)\]

   Odd
   \(f(-x) = -f(x)\)

15. State yes or no if the following graphs show a function that has an inverse?
   a) Yes
   b) No
   c) Yes
16. Find the inverse of each function.
   a) \( y = x^3 + 8 \)
      \[ x = y^3 + 8 \]
      \[ x - 8 = y^3 \]
      \[ y = \sqrt[3]{x - 8} \]
   b) \( f(x) = x^2 - 3 \)
      \[ x = y^2 - 3 \]
      \[ x + 3 = y^2 \]
      \[ \sqrt{x + 3} = y \]
      \[ \ln x = \ln 2^y \]
      \[ \ln x = y \ln 2 \]
      \[ \ln x = \frac{y}{\ln 2} \]
      \[ y = \log_2 x \]
   c) \( y = 2^x \)
      \[ x = 2^y \]

17. Simplify the logarithms.
   a) \( \log 10 + e \ln e \)
      \[ \frac{1 + e \cdot 1}{1 + e} \]
   b) \( 10^{-3(\log B)} \)
      \[ (10 \log B)^{-3} \]
      \[ B^{-3} \]
      \[ \frac{1}{B^3} \]
      \[ \text{or} \]
      \[ B^3 \]

18. Solve for \( x \). Write answers in exact form.
   a) \( \ln x^2 = 10 \)
      \[ e^{10} = x^2 \]
      \[ \pm \sqrt{e^{10}} = x \]
      \[ \pm e^5 = x \]
   b) \( 4 \cdot 6^x = 8^x \)
      \[ \ln 4 \cdot 6^x = \ln 8^x \]
      \[ \ln 4 + x \ln 6 = x \ln 8 \]
      \[ \ln 4 + \ln 6^x = x \ln 8 \]
      \[ \ln 4 = x \ln 8 - x \ln 6 \]
      \[ \ln 4 = x(\ln 8 - \ln 6) \]
      \[ x = \frac{\ln 4}{\ln 8 - \ln 6} \]
      \[ \text{or} \]
      \[ \log_3 4 \]
19. Find the value of each expression.
   a) \( \cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2} \)
   b) \( \sin \left(-\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2} \)
   c) \( \tan \frac{5\pi}{6} = -\frac{\sqrt{3}}{3} \)
   d) \( \csc \frac{\pi}{3} = \frac{2}{\sqrt{3}} \)
   e) \( \sec \left(-\frac{\pi}{6}\right) = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \)
   f) \( \cot \left(-\frac{19\pi}{4}\right) = 1 \)

20. Find the value of each expression.
   a) \( \arcsin \left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} \)
   b) \( \arcsin \left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \)
   c) \( \arccos \left(0\right) = \frac{\pi}{2} \)
   d) \( \arccos \left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6} \)
   e) \( \arcsin \left(-\frac{1}{2}\right) = -\frac{\pi}{6} \)
   f) \( \arctan \left(\sqrt{3}\right) = \frac{\pi}{3} \)

21. a) Find a possible formula for the graph.
   \[ y = -4\cos \frac{2}{3}(x - \frac{\pi}{2}) + 1 \]
   b) Sketch the graphs of the functions.
   Sketch one period from the “starting point.”
   \[ y = \sin 2(x + \pi) + 3 \]
22. Solve for $\theta$ for $0 \leq \theta \leq 2\pi$.

a) $8 \cos^2 \theta - 4 = 0$

\[ 8 \cos^2 \theta = 4 \]
\[ \cos^2 \theta = \frac{1}{2} \]
\[ \cos \theta = \pm \frac{\sqrt{2}}{2} \]
\[ \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \]

b) $2 \sin^2 x = 2 - 3 \sin x$

\[ 2 \sin^2 x + 3 \sin x - 2 = 0 \]
\[ (2 \sin x - 1)(\sin x + 2) = 0 \]
\[ 2 \sin x - 1 = 0 \quad \sin x + 2 = 0 \]
\[ \sin x = \frac{1}{2} \quad \sin x = -2 \]
\[ x = \frac{\pi}{6}, \frac{5\pi}{6} \]

c) $2 \cos 2\theta + 1 = 0$

\[ \cos 2\theta = -\frac{1}{2} \]
\[ 2\theta = \pm \frac{2\pi}{3}, \pm \frac{4\pi}{3} \]
\[ \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \]

If $2\theta$, then find all answers for two rotations.

\[ 2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3} \]
\[ \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3} \]

d) $e^x \cos x - e^x \sin x = 0$

\[ e^x (\cos x - \sin x) = 0 \]
\[ e^x = 0 \quad \cos x = \sin x = 0 \]
\[ x = \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \]

Another method: sub in $\cos^2 x + \sin^2 x = 1$

\[ 2 \cos^2 x = \frac{1}{2} \]
\[ \cos x = \pm \frac{1}{\sqrt{2}} \]
\[ \cos x = \pm \frac{\sqrt{2}}{2} \]
\[ x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \]
23. Consider the polynomial \( f(x) = x^3 - 8x^2 + 16x \).

a) Find the zeros.

\[
\begin{align*}
x^3 - 8x^2 + 16x &= 0 \\
x(x^2 - 8x + 16) &= 0 \\
x = 0 \\
(x - 4)^2 &= 0 \\
x &= 4
\end{align*}
\]

b) Classify the zeros as single, double or triple.

\[
\begin{align*}
x = 0 \text{ single} \\
x = 4 \text{ double}
\end{align*}
\]

\[
\begin{array}{c}
0 < x < 4, x > 4 \Rightarrow \frac{1}{4} \times x = \text{ not possible}
\end{array}
\]

c) What interval(s) of \( f(x) \) is positive?

\[
\begin{array}{c}
\text{since } x = 0 \text{ and } x = 4
\end{array}
\]

24. Write the equation of each graph. Leave answer in factored form.

Hint: Use the form \( y = k(x - r_1)^a(x - r_2)^b(x - r_3)^c \ldots \)

Find the roots, then substitute one of the coordinates to find \( k \).

\[
y = k(x + 5)(x + 1)(x - 2)(x - 5)
\]

Plug in \((0, 3)\)

\[
2 = k(5)(1)(-2)(-5)
\]

\[
\frac{1}{25} = k
\]

\[
y = \frac{1}{25}(x + 5)(x + 1)(x - 2)(x - 5)
\]

25. Solve the following equation. \( x^{10} - 1 = 3^x \)

Solve by Graphing!

\[
\begin{align*}
x &= -1.028393 \\
x &= 1.1647441 \\
x &= 31.863656
\end{align*}
\]

But as \( x \to +\infty \)