1. For what value(s) of $x$ is \( f(x) = \frac{x}{x^2 - 1} \) discontinuous?

\[
x^2 - 1 = 0
\]

\[
x = \pm 1
\]

2. \( f(x) = \frac{x}{x^2 + 2x} \) is continuous for all real numbers EXCEPT:

\[
x^2 + 2x = 0
\]

\[
x(x + 2) = 0
\]

\[
x = 0, x = -2
\]

3. \( f(x) = \frac{x + 2}{x^2 - 4} \) is undefined at

\[
x^2 - 4 = 0
\]

\[
x = \pm 2
\]

4. \( f(x) = \frac{1}{x^2 + 1} \) is defined for all real numbers EXCEPT:

\[
x^2 + 1 = 0
\]

\[
x^2 = -1
\]

\[
x = \pm \sqrt{-1}
\]

For #5 to #8, find the vertical asymptotes of each function.

5. \( f(x) = \frac{x}{x^2 - 49} \)

\[
x^2 - 49 = 0
\]

\[
x = 7, x = -7
\]

6. \( f(x) = \frac{x}{4x + 8} \)

\[
4x + 8 = 0
\]

\[
x = -2
\]

7. \( f(x) = \frac{x^2 - x}{x - 1} \)

No V.A.

8. \( f(x) = \frac{x^3 + 2x^2 - x - 2}{x^3 + x^2 - 2x} \)

\[
x^2(x + 2) - 1(x + 2)
\]

\[
x(x^2 + x - 2)
\]

\[
(x + 2)(x^2 - 1)
\]

\[
x(x + 2)(x - 1)
\]

\[
x(x + 2)(x + 1)(x - 1)
\]

\[
x(x + 2)(x + 1)(x - 1)
\]

\[
x = 0
\]
For #9 to #12, find the horizontal asymptotes of each function.

9. \( f(x) = \frac{x^2 - 9}{3x + 2} \)  
   \[ \text{NO H.A.} \]

10. \( f(x) = \frac{x^2 - 4x + 4}{4x^2 - 1} \)  
    \[ y = \frac{1}{4} \]

11. \( f(x) = \frac{x}{x^3 - 2} \)  
    \[ y = 0 \]

12. \( f(x) = \frac{x^3 - 1}{x - 1} \)  
    \[ \frac{(x-1)(x^2 + x + 1)}{(x-1)} \]
    \[ \text{NO H.A.} \]

Multiple Choice.

13. \( f(x) = \frac{(x-1)^2}{x^2 - 1} \) has  
    \[ \frac{(x-1)(x-1)}{(x+1)(x-1)} \]
    \[ \text{VA } x = -1 \text{ Hole } x = 1 \]
   (A) a hole at \( x = -1 \)  
   (B) holes at \( x = -1 \) and \( x = 1 \)  
   (C) vertical asymptotes at \( x = 1 \) and \( x = -1 \)  
   (D) horizontal asymptote at \( y = -1 \)  
   (E) a hole at \( x = 1 \) and a vertical asymptote at \( x = -1 \)

14. For \( f(x) = \frac{x^3 + 8}{(x+2)^2} \), choose all that are true.  
   \[ \frac{(x+2)(x^2 - 2x + 4)}{(x+2)(x+2)} \]
   \[ x = -2 \text{ Hole } x = -2 \text{ VA } \]
   \[ \text{Can't be both!!} \]
   (A) hole at \( x = -2 \)  
   (B) vertical asymptote at \( x = -2 \)  
   (C) horizontal asymptote at \( y = 0 \)  
   (D) horizontal asymptote at \( y = 1 \)  
   (E) no horizontal asymptote
Find each characteristic of the rational functions and then sketch their graphs.

15. $y = \frac{5x}{x-2}$

a) zeros

\[
\begin{align*}
5x &= 0 \\
x &= 0
\end{align*}
\]

b) y-intercepts

\[
\begin{align*}
y &= \frac{5(0)}{0-2} \\
y &= 0
\end{align*}
\]

d) vertical asymptotes

\[
\begin{align*}
x-2 &= 0 \\
x &= 2
\end{align*}
\]

\[
\begin{align*}
\text{same} \\
\text{x = 2}
\end{align*}
\]

e) undefined values of x

f) horizontal or slant asymptotes

\[
y = 5
\]

\[
\text{essential at } x = 2
\]

\[
\text{graph}
\]
16. \( y = \frac{2x}{x^2 - 1} \)

a) zeros

\[
0 = \frac{2x}{x^2 - 1} \\
0 = 2x \\
0 = x
\]

b) \( y \)-intercepts

\[
y = \frac{2(0)}{0^2 - 1} \\
y = 0
\]

c) undefined values of \( x \)

\[
x^2 - 1 = 0 \\
x = \pm 1
\]

d) vertical asymptotes

\[
x = \pm 1
\]

e) horizontal or slant asymptotes

\[
y = 0
\]

f) identify and locate all discontinuities

\[
\text{essential at } x = \pm 1
\]
17. \( y = \frac{x^2}{x-3} \)

a) zeros
\[
\begin{align*}
x = 0 & \\
\square & \\
x = 0 & 
\end{align*}
\]

b) \( y \)-intercepts
\[
\begin{align*}
y = 0 & \\
\square & \\
y = 0 & 
\end{align*}
\]

c) undefined values of \( x \)
\[
\begin{align*}
(x-3) = 0 & \\
x = 3 & \rightarrow \text{same} \\
\square & \\
x = 3 & 
\end{align*}
\]

d) vertical asymptotes
\[
\begin{align*}
y = 0 & \\
\square & \\
y = 0 & 
\end{align*}
\]

e) horizontal or slant asymptotes
No HA.

Slant
\[
\begin{align*}
\frac{x^2}{x-3} + \frac{9}{x-3} & \\
\frac{x^3}{x-3} & \\
- \left( x^2 - 3x \right) & \\
- \frac{3x}{x-3} & \\
\frac{3x}{x-3} & \\
\frac{9}{x-3} & 
\end{align*}
\]

f) identify and locate all discontinuities
Essential at \( x = 3 \)

\[
\begin{align*}
a \quad x \to \infty \quad \Rightarrow \\
y = x + 3 + \frac{9}{x-3} & \rightarrow \quad y = x + 3 
\end{align*}
\]
18. \[ y = \frac{x^2 - 1}{x - 1} \]

a) zeros
\[ \frac{x+1}{x-1} = 0 \]
\[ x + 1 = 0 \]
\[ x = -1 \]

b) \( y \)-intercepts
\[ y = \frac{0^2 - 1}{0 - 1} \]
\[ y = -1 \]

\[ \text{None} \]

c) undefined values of \( x \)
\[ \frac{x - 1}{x - 1} = 0 \]
\[ x - 1 = 0 \]
\[ x = 1 \]

d) vertical asymptotes

\[ \text{None} \]

e) horizontal or slant asymptotes

\[ \text{No HA} \]
\[ \text{No SA} \]

since \( \frac{x^2 - 1}{x - 1} \rightarrow x + 1 \)

\[ \text{\( x \)-graph with a hole at} \]
\[ x = 1 \]

f) identify and locate all discontinuities

\[ \text{Removable at} \ x = 1 \]