The Derivative – Day 4 WKST

Name:_____

Interpreting the Derivative
Interpreting a Limit as a Derivative

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1. Suppose that at a price of \$*p*, a quantity, *q*, of the commodity is sold. If $q = f(p)$, explain in economic terms the meaning of the following.

   a) $f(10) = 240{,}000$

   b) $f'(10) = 240{,}000$

   c) $f^{-1}(10) = 240{,}000$

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2. After investing \$1000 at an annual interest rate of 7% compounded continuously for *t* years, your balance is \$*B*, where $B = f(t)$.

   a) What are the units of $\frac{dB}{dt}$?

   b) What is the financial interpretation of $\frac{dB}{dt}$?

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3. After investing \$1000 at an annual interest rate of *r* % compounded continuously for 7 years, your balance is \$*B*, where $B = f(r)$.

   a) What are the units of $\frac{dB}{dr}$?

   b) What is the financial interpretation of $\frac{dB}{dr}$?

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4. The temperature, T, in degrees Fahrenheit, of a cold yam placed in a hot oven (set at 350º) is given by $T = f(t)$, where *t* is the time in minutes since the yam was put in the oven.

   a) What is the sign of $f'(t)$?

   b) What are the units of $f'(20)$?

   c) What is the practical meaning of $f'(20) = 2$?

5. If $\lim_{x\to\infty} f(x) = 50$ and $f'(x)$ is positive for all *x*, what is $\lim_{x\to\infty} f'(x)$? (Assume this limit exists.) Explain your answer with a possible graph of $f(x)$.

6. A company's revenue from car sales, C (measured in thousands of dollars) is a function of advertising expenditure, *a*, also measured in thousands of dollars. Suppose $C = f(a)$.

   a) What does the company hope is true about the sign of f'?

   b) What does the statement $f'(100) = 2$ mean in practical terms?

   c) What does the statement $f'(100) = .5$ mean in practical terms?

   d) Suppose the company plans to spend about $100,000 on advertising.

      i) If $f'(100) = 2$, should the company spend slightly more or slightly less than $100,000 on advertising?

      ii) If $f'(100) = .5$, should the company spend slightly more or slightly less than $100,000 on advertising?

   e) In general, what value does the company hope f' is greater than?

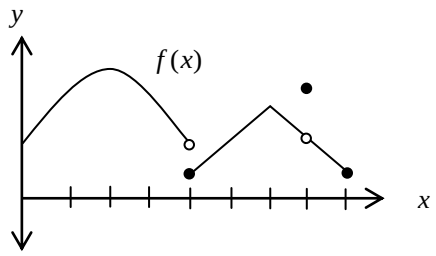
7. Evaluate the following limits.

   a) $\lim_{h\to 0} \dfrac{(2+h)^3 - 2^3}{h} =$

   b) $\lim_{h\to 0} \dfrac{\sqrt{9+h} - \sqrt{9}}{h} =$

c) $\lim_{h\to 0} \dfrac{\dfrac{1}{4+h}-\dfrac{1}{4}}{h} =$

8. In the diagram below, f is a piece-wise function with a horizontal tangent at $x = 2$. Determine whether each statement is true or false.



(A) $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$

(B) $f(x)$ is continuous at $x = 3$

(C) $\lim_{x\to 3} f(x) = f(3)$

(D) $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = 0$

(E) $\lim_{h\to 0} \dfrac{f(4+h)-f(4)}{h}$ exists

(F) $\lim_{x\to 2} f(x) = 0$

(G) $\lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h} > 0$

(H) $\lim_{x\to 7} f(x)$ is nonexistent

(I) $\lim_{h\to 0} \dfrac{f(6+h)-f(6)}{h}$ is nonexistent

(J) $\lim_{h\to 0} \dfrac{f(7+h)-f(7)}{h}$ is nonexistent

Answers

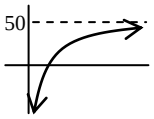
1. a) 240,000 sold when the price is \$10
   b) at a price of \$10, expect an increase of 240,000 sold when the price increases
   c) 10 sold when the price is \$240,000

2. a) \$/yr b) the expected increase in your balance per year

3. a) \$/% b) the expected increase in your balance for an increase of 1% in your APR

4. a) $f'(t) > 0$ since the temperature is increasing
   b) degree/minute
   c) after 20 minutes, you should expect an increase of 2°

5. $\lim_{x\to\infty} f'(x) = 0$?



6. a) $f' > 0$
   b) at \$100,000 of advertising expense, expect an increase of \$2 for every additional dollar spent
   c) at \$100,000 of advertising expense, expect an increase of \$1 for every \$2spent OR expect an increase of 50 cents for every additional dollar spent
   d) Suppose the company plans to spend about \$100,000 on advertising.
      i) more
      ii) less
   e) it's more desirable for $f' > 1$

7. a) 12
   b) 1/6
   c) −1/16

8. a) False
   b) True
   c) True
   d) True
   e) False
   f) False
   g) True
   h) False
   i) True
   j) True