1. Suppose that at a price of $p$, a quantity, $q$, of the commodity is sold. If $q = f(p)$, explain in economic terms the meaning of the following.

a) $f(10) = 240,000$

b) $f'(10) = 240,000$

c) $f^{-1}(10) = 240,000$

2. After investing $1000 at an annual interest rate of 7% compounded continuously for $t$ years, your balance is $B$, where $B = f(t)$.

a) What are the units of $\frac{dB}{dt}$?

b) What is the financial interpretation of $\frac{dB}{dt}$?

3. After investing $1000 at an annual interest rate of $r\%$ compounded continuously for 7 years, your balance is $B$, where $B = f(r)$.

a) What are the units of $\frac{dB}{dr}$?

b) What is the financial interpretation of $\frac{dB}{dr}$?

4. The temperature, $T$, in degrees Fahrenheit, of a cold yam placed in a hot oven (set at 350º) is given by $T = f(t)$, where $t$ is the time in minutes since the yam was put in the oven.

a) What is the sign of $f'(t)$?

b) What are the units of $f'(20)$?

c) What is the practical meaning of $f'(20) = 2$?
5. If \( \lim_{x \to \infty} f(x) = 50 \) and \( f'(x) \) is positive for all \( x \), what is \( \lim_{x \to \infty} f'(x) ? \) (Assume this limit exists.)

Explain your answer with a possible graph of \( f(x) \).

6. A company’s revenue from car sales, \( C \) (measured in thousands of dollars) is a function of advertising expenditure, \( a \), also measured in thousands of dollars. Suppose \( C = f(a) \).

a) What does the company hope is true about the sign of \( f' \)?

b) What does the statement \( f'(100) = 2 \) mean in practical terms?

c) What does the statement \( f'(100) = .5 \) mean in practical terms?

d) Suppose the company plans to spend about $100,000 on advertising.

i) If \( f'(100) = 2 \), should the company spend slightly more or slightly less than $100,000 on advertising?

ii) If \( f'(100) = .5 \), should the company spend slightly more or slightly less than $100,000 on advertising?

e) In general, what value does the company hope \( f' \) is greater than?

7. Evaluate the following limits.

a) \( \lim_{h \to 0} \frac{(2+h)^3 - 2^3}{h} = \)

b) \( \lim_{h \to 0} \frac{\sqrt{9+h} - \sqrt{9}}{h} = \)
c) \[
\lim_{h \to 0} \frac{1}{4 + h} - \frac{1}{h} =
\]

8. In the diagram below, \( f \) is a piece-wise function with a horizontal tangent at \( x = 2 \).
Determine whether each statement is true or false.

(A) \[
\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)
\]

(B) \( f(x) \) is continuous at \( x = 3 \)

(C) \[
\lim_{x \to 3} f(x) = f(3)
\]

(D) \[
\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} = 0
\]

(E) \[
\lim_{h \to 0} \frac{f(4 + h) - f(4)}{h} \text{ exists}
\]

(F) \[
\lim_{x \to 2} f(x) = 0
\]

(G) \[
\lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} > 0
\]

(H) \( f(x) \) is nonexistent at \( x = 7 \)

(I) \[
\lim_{h \to 0} \frac{f(6 + h) - f(6)}{h} \text{ is nonexistent}
\]

(J) \[
\lim_{h \to 0} \frac{f(7 + h) - f(7)}{h} \text{ is nonexistent}
\]
**Answers**

1. a) 240,000 sold when the price is $10  
   b) at a price of $10, expect an increase of 240,000 sold when the price increases  
   c) 10 sold when the price is $240,000

2. a) $/yr  
   b) the expected increase in your balance per year

3. a) $/%  
   b) the expected increase in your balance for an increase of 1% in your APR

4. a) \( f'(t) > 0 \) since the temperature is increasing  
   b) degree/minute  
   c) after 20 minutes, you should expect an increase of 2°

5. \( \lim_{x \to \infty} f'(x) = 0 \)

6. a) \( f' > 0 \)  
   b) at $100,000 of advertising expense, expect an increase of $2 for every additional dollar spent  
   c) at $100,000 of advertising expense, expect an increase of $1 for every $2 spent OR expect an increase of 50 cents for every additional dollar spent  
   d) Suppose the company plans to spend about $100,000 on advertising.  
      i) more  
      ii) less  
   e) it’s more desirable for \( f' > 1 \)

7. a) 12  
   b) 1/6  
   c) -1/16

8. a) False  
   b) True  
   c) True  
   d) True  
   e) False  
   f) False  
   g) True  
   h) False  
   i) True  
   j) True