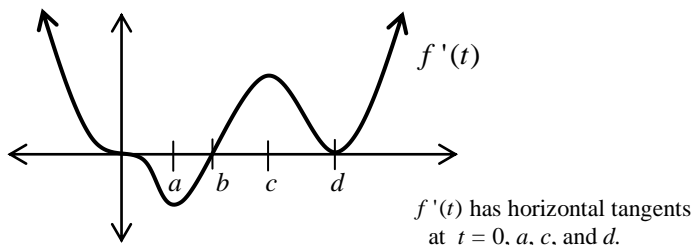


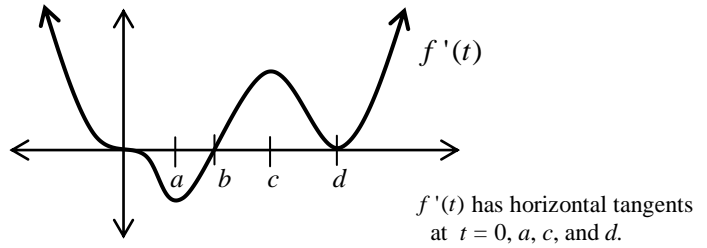
The Second Derivative

Use the figure on the right to answer #1 to #13.
Notice that **THE DERIVATIVE IS SHOWN**.



1. What are the critical points of $f(t)$?
 - (A) $0, a, c$
 - (B) a, c
 - (C) $0, b, d$
 - (D) a, c, d
 - (E) $0, a, c, d$
2. When is $f(t)$ a maximum?
 - (A) a
 - (B) b
 - (C) c
 - (D) d
 - (E) 0
3. When is $f(t)$ a minimum?
 - (A) a
 - (B) b
 - (C) c
 - (D) d
 - (E) 0
4. Which critical points is neither a maximum nor minimum value of $f(t)$?
 - (A) a
 - (B) b
 - (C) c
 - (D) d
 - (E) 0
5. When is $f''(t) = 0$?
 - (A) $0, a, c$
 - (B) a, c
 - (C) $0, b, d$
 - (D) a, c, d
 - (E) $0, a, c, d$
6. Where does $f(t)$ have points of inflection?
 - (A) $0, a, c$
 - (B) a, c
 - (C) $0, b, d$
 - (D) a, c, d
 - (E) $0, a, c, d$
7. When is $f'(t)$ increasing?
 - (A) $(-\infty, a)$ and (c, d)
 - (B) (a, c) and $(d, +\infty)$
 - (C) $(-\infty, 0), (b, +\infty)$
 - (D) $(-\infty, 0), (b, d),$ and $(d, +\infty)$
 - (E) $(0, b)$
8. When is $f'(t)$ decreasing?
 - (A) $(-\infty, a)$ and (c, d)
 - (B) (a, c) and $(d, +\infty)$
 - (C) $(-\infty, 0), (b, +\infty)$
 - (D) $(-\infty, 0), (b, d),$ and $(d, +\infty)$
 - (E) $(0, b)$
9. When is $f''(t)$ positive?
 - (A) $(-\infty, a)$ and (c, d)
 - (B) (a, c) and $(d, +\infty)$
 - (C) $(-\infty, 0), (b, +\infty)$
 - (D) $(-\infty, 0), (b, d),$ and $(d, +\infty)$
 - (E) $(0, b)$
10. When is $f''(t)$ negative?
 - (A) $(-\infty, a)$ and (c, d)
 - (B) (a, c) and $(d, +\infty)$
 - (C) $(-\infty, 0), (b, +\infty)$
 - (D) $(-\infty, 0), (b, d),$ and $(d, +\infty)$
 - (E) $(0, b)$

The figure on the right is the same as the front page.



11. When is $f(t)$ concave up?

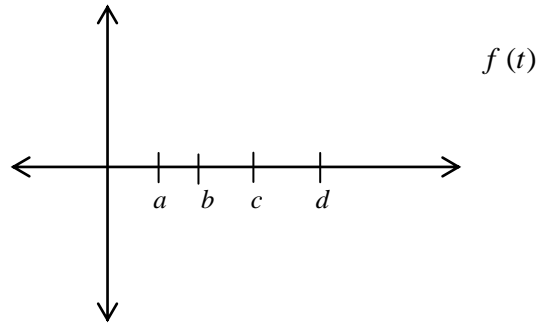
- (A) $(-\infty, a)$ and (c, d)
- (B) (a, c) and $(d, +\infty)$
- (C) $(-\infty, 0), (b, +\infty)$
- (D) $(-\infty, 0), (b, d),$ and $(d, +\infty)$
- (E) $(0, b)$

12. When is $f(t)$ concave down?

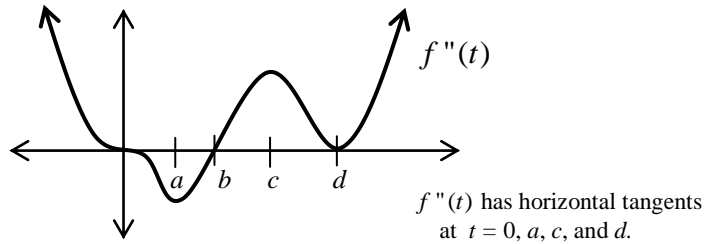
- (A) $(-\infty, a)$ and (c, d)
- (B) (a, c) and $(d, +\infty)$
- (C) $(-\infty, 0), (b, +\infty)$
- (D) $(-\infty, 0), (b, d),$ and $(d, +\infty)$
- (E) $(0, b)$

13. Sketch a graph of $f(t)$.

Suppose $f(0) = 0$.



Use the figure on the right to answer #14 to #16. Notice that **THE SECOND DERIVATIVE IS SHOWN**.



14. When is $f''(t) = 0$?

- (A) $0, a, c$
- (B) a, c
- (C) $0, b, d$
- (D) a, c, d
- (E) $0, b$

15. Where does $f(t)$ have points of inflection?

- (A) $0, a, c$
- (B) a, c
- (C) $0, b, d$
- (D) a, c, d
- (E) $0, b$

16. What are the critical points of $f(t)$?

- (A) $0, a, c$
- (B) a, c
- (C) $0, b, d$
- (D) a, c, d
- (E) cannot determine

17. For all x in the closed interval $[2, 5]$, identify the signs (ie positive or negative) of the first and second derivatives of each table?

(A)

| x | $f(x)$ |
|-----|--------|
| 2 | 7 |
| 3 | 9 |
| 4 | 12 |
| 5 | 16 |

$f'(x):$ _____

$f''(x):$ _____

(B)

| x | $f(x)$ |
|-----|--------|
| 2 | 7 |
| 3 | 12 |
| 4 | 15 |
| 5 | 16 |

$f'(x):$ _____

$f''(x):$ _____

(C)

| x | $f(x)$ |
|-----|--------|
| 2 | 16 |
| 3 | 13 |
| 4 | 10 |
| 5 | 7 |

$f'(x):$ _____

$f''(x):$ _____

Use the following table to answer #18 to #22.

$f(t)$ represents the height, in feet, of an object projected upward after t seconds.

| | | | | |
|----------------|---|----|-----|-----|
| Time (seconds) | 0 | 1 | 2 | 3 |
| Height (feet) | 6 | 74 | 110 | 114 |

18. Estimate $f'(1)$.

19. Estimate $f'(2)$.

20. Use the results of #19 and #20 to estimate $f''(1)$.

Suppose $f(t) = -16t^2 + 84t + 6$ is the function modeled by the previous table.

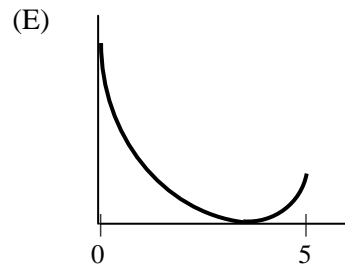
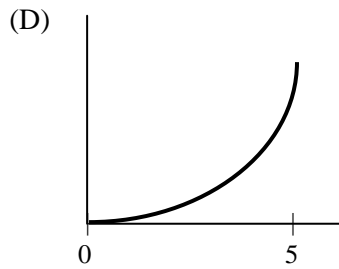
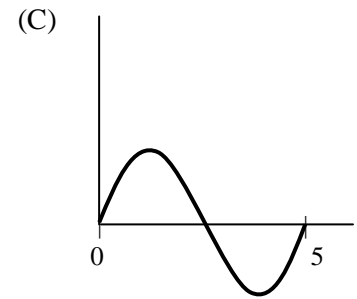
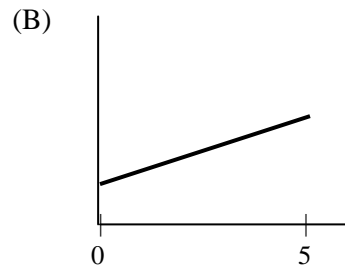
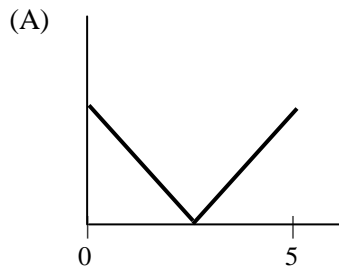
21. Use the short-cut for derivatives to find $f''(1)$.

Compare your answer to #21.

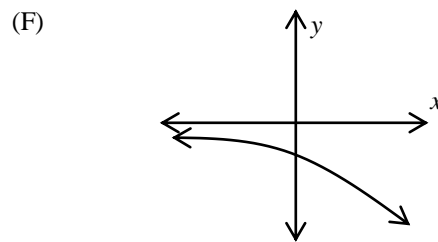
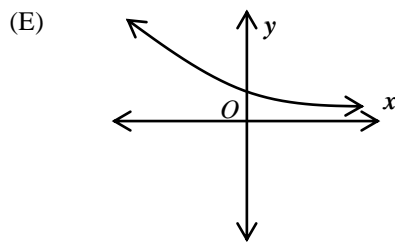
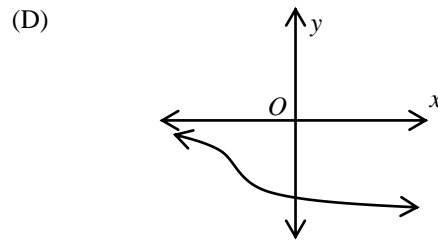
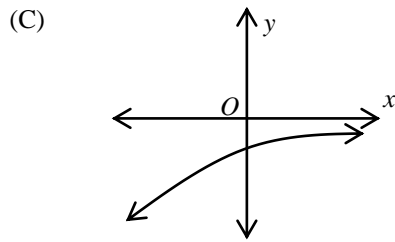
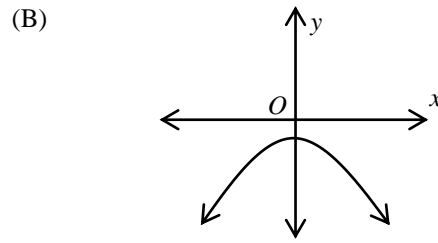
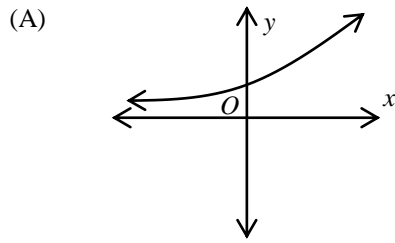
22. What are the units and the practical meaning of $f''(1)$?

Each of the graphs shows the position of a particle moving along the x -axis as a function of time, $0 \leq t \leq 5$. The vertical scale of the graphs is the same. Match the graphs with the following descriptions. More than one answer is possible.

- 29. constant velocity
- 30. the greatest initial velocity
- 31. the greatest average velocity
- 32. zero average velocity
- 33. zero acceleration throughout
- 34. positive acceleration throughout



35. Which of the following graphs of f below would $f'(x) \cdot f''(x) < 0$? More than one answer is possible.



ANSWERS:

- | | | | | | | | |
|------|-------|-----------|-----------------|--|-----------------------------|----------|----------|
| 1) C | 6) D | 11) B | 16) E | 19) 20 | 23) A | 28) C | 33) B |
| 2) E | 7) B | 12) A | 17A) pos, pos | 20) -32 | 24) incr in larger amounts | 29) B | 34) D, E |
| 3) B | 8) A | 13) graph | B) pos, neg | 21) -32 | 25) incr in smaller amounts | 30) C | 35) C, E |
| 4) D | 9) B | 14) C | C) neg, neither | 22) ft/sec ² , grav accl | 26) decr in smaller amounts | 31) D | |
| 5) E | 10) A | 15) E | 18) 52 | | 27) decr in larger amounts | 32) A, C | |