Ch 1 / The Derivative TEST – Calc AB   Name:____________________   PT

Practice

Part A   **No Calculator**

1. The position of a particle on the *x*-axis, *x* (*t*), is given at one second intervals by the following table for $0 \le t \le 4$.

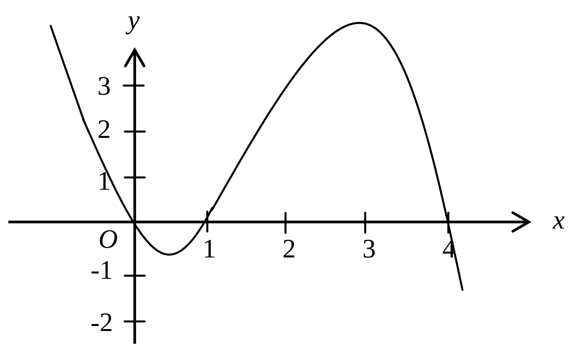
t	0	1	2	3	4
$x(t)$	10	15	11	2	-2

The average velocity for the particle over this time interval is

(A) -4   (B) -3   (C) 1.6   (D) 3   (E) 4

2. The power rule tells us that $\lim_{h\to 0} \frac{(2+h)^{17}-2^{17}}{h} =$

(A) 0   (B) 2^{17} (C) $2^{17}\ln(2)$ (D) 2^{16} (E) $17\cdot 2^{16}$



3. The graph above is the graph of $y = f(x)$. At which of the following points is $f'(x)$ the greatest?

(A) $x = -1$ (B) $x = 0$ (C) $x = 2$ (D) $x = 3$ (E) $x = 4$

Part A **No Calculator**

4. If f is an increasing function whose graph lies below the x-axis and is concave up, then which of the following must be true?

I. $f(x) < 0$
II. $f'(x) < 0$
III. $f''(x) < 0$

(A) I only (B) II only (C) III only
(D) I and II only (E) I and III only

---

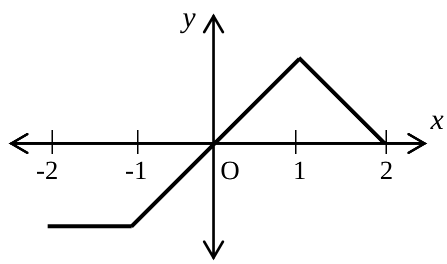
5. If $f(x) = 12\sqrt{x}$, then $f'(x) = \frac{6}{\sqrt{x}}$. This means that an equation for the line tangent to the graph of f when $x = 9$ is

(A) $y = 36$ (B) $y = 2$ (C) $y = 36(x - 9)$
(D) $y = 2(x - 9)$ (E) $y = 2x + 18$

---

6. If $f(x) = 2x^3 + \frac{1}{x^2}$, then $f'(2) =$

(A) 12 (B) 12.5 (C) 23.75 (D) 24 (E) undefined



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

(A) f is increasing for $-1 \le x \le 1$
(B) f is decreasing for $-1 \le x \le 2$
(C) f is increasing for $-2 \le x \le 1$
(D) f is decreasing for $-2 \le x \le 0$
(E) f is not differentiable at $x = -1$ and $x = 1$

---

8. If $f(x) = \begin{cases} 2x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$, then $f'(0)$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) undefined

---

9. If $f(x) = \begin{cases} 6x+1, & \text{if } x < 2 \\ 9, & \text{if } x = 2 \\ 6x-4, & \text{if } x > 2 \end{cases}$, then $f'(2)$ is

(A) 0 (B) 6 (C) 9 (D) 12 (E) undefined

10. If $f(x)=\begin{cases}\dfrac{x^2-9}{x-3}, & \text{if } x\neq 3\\ 6\ , & \text{if } x=3\end{cases}$, then $f'(3)$ is

(A) 0 (B) 1 (C) 3 (D) 6 (E) undefined

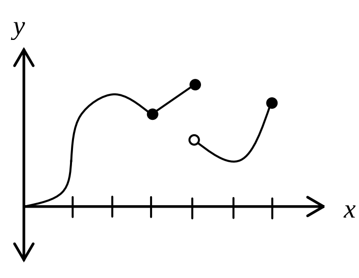
11. A table of values for $f(t)$ is shown below.

t	0	1	2	3	4
$f(t)$	0	2	6	12	20

According to the table, the best approximation for $f'(2)$ is

(A) 5 (B) 10 (C) 6 (D) 12 (E) 8

12. In the diagram below, f has a vertical tangent at $x = 1$ and horizontal tangents at $x = 2$ and at $x = 5$. All of these statements are true EXCEPT



(A) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x)$

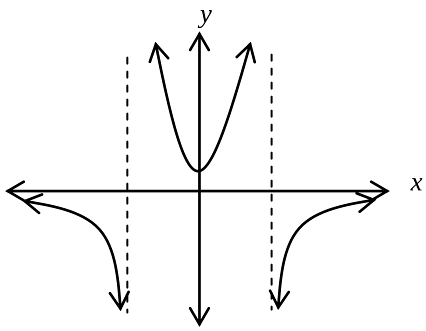
(B) $\lim_{x\to 5} f(x) = f(5)$

(C) $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=0$

(D) $\lim_{h\to 0^-}\dfrac{f(4+h)-f(4)}{h} = \lim_{h\to 0^+}\dfrac{f(4+h)-f(4)}{h}$

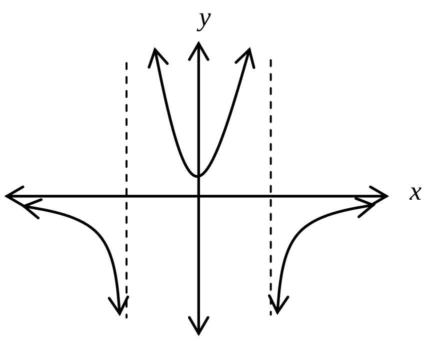
(E) $\lim_{x\to 2.5} f(x) > \lim_{h\to 0}\dfrac{f(2.5+h)-f(2.5)}{h}$

13. The graph of the function f is given below.

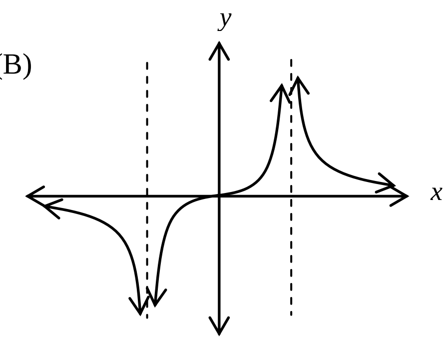


Which of these graphs could be the derivative of f?

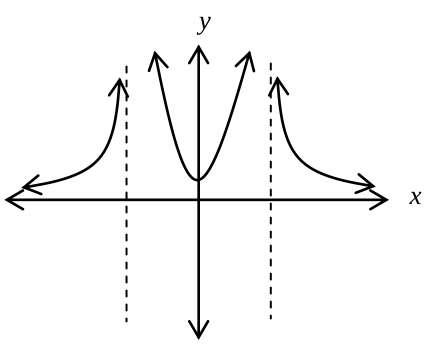
(A)



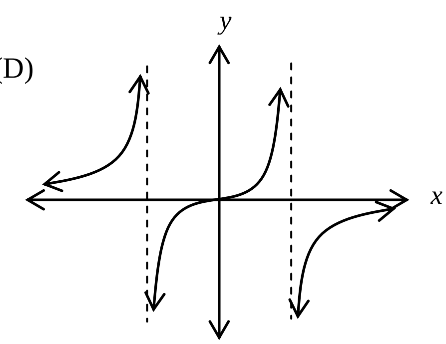
(B)



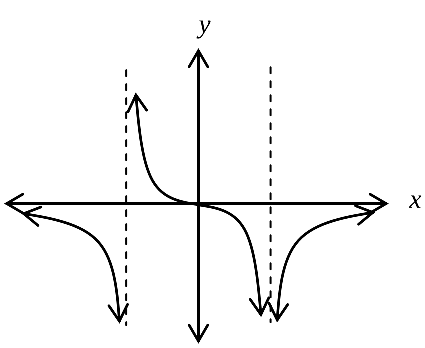
(C)



(D)



(E)



14. If $f(1)=5$ is a point on $f(x)$ and the line tangent at that point passes through (-2 , 4), then $f'(1)=$

(A) 1 (B) $\frac{1}{3}$ (C) -2 (D) $\frac{1}{2}$ (E) 3

15.

x	0	.5	1	1.5	2
$f(x)$	0	2.3	1.4	1.1	1.5

According to the table of $f(x)$ above, what is the equation of the tangent line at $x=1$?

(A) $y=-\frac{5}{6}x+\frac{7}{6}$ (B) $y=-\frac{5}{6}x+\frac{67}{30}$ (C) $y=\frac{5}{6}x+\frac{17}{30}$

(D) $y=-\frac{6}{5}x+\frac{13}{5}$ (E) $y=-\frac{6}{5}x+\frac{11}{5}$

---

16. For a twice-differentiable function f on the closed interval [1 , 4] , $f'(x)$ and $f''(x)$ have the same sign. Which of the following could be a table of values for f?

(A)

x	y
1	10
2	7
3	4
4	1

(B)

x	y
1	15
2	7
3	3
4	1

(C)

x	y
1	1
2	4
3	7
4	10

(D)

x	y
1	1
2	9
3	13
4	15

(E)

x	y
1	15
2	13
3	9
4	1

Stop! You may use your graphing calculator for the remainder of the test.

17. The table below shows selected values of x, $f(x)$, and $f'(x)$.

x	0	1	2	3	4
$f(x)$	0	3	18	57	132
$f'(x)$	1	7	25	55	97

The best approximation for $f''(2)$ is

(A) 16 (B) 20 (C) 24 (D) 32 (E) 36

18. If $V(t)$ represents the number of thousands of gallons of water in a tank t hours after midnight on a fixed day, then which of the following pairs of equations can be used to express the statement, “At 3 PM there were 9000 gallons of water in the tank, and the amount of water in the tank was decreasing at the rate of 200 gallons per hour”?

(A) $V(3) = 9000$ and $V'(3) = -200$

(B) $V(3) = 9$ and $V'(3) = 200$

(C) $V(15) = 9000$ and $V'(15) = -200$

(D) $V(15) = 9$ and $V'(15) = -0.2$

(E) $V(15) = 9$ and $V'(15) = 0.2$

Part B **Graphing Calculator Required**

19. If $P(t)$ is the population in millions, of a country t years after 1900, then what does the equation $(P^{-1})'(60) = 10$ mean?

(A) In 1960, the population was growing at the rate of 10 million people per year.
(B) In 1910, the population was growing at the rate of 60 million people per year.
(C) In 1960, the population was growing at the rate of $\frac{1}{10}$ million people per year.
(D) When the population was 60 million, it was growing at the rate of 10 million people per year.
(E) When the population was 60 million, it was growing at the rate of $\frac{1}{10}$ million people per year.

20. If the graph of f goes through the points (2 , 5), (4 , 12), and (8 , 22), and neither $f'(x)$ nor $f''(x)$ changes sign on the interval (2 , 8), then the graph must be

(A) increasing and concave up (B) increasing and concave down (C) decreasing and concave up
(D) decreasing and concave down (E) none of the above.

21. If the graph of f goes through the points (1 , 4), (2 , 6), and (3 , 10), and neither $f'(x)$ nor $f''(x)$ changes sign on the interval (1 , 3), then the graph of f must be

(A) increasing and concave up (B) increasing and concave down (C) decreasing and concave up
(D) decreasing and concave down (E) none of the above.

Part B **Graphing Calculator Required**

22. If $f(p) = q$ means that, at a price of p dollars, a sandwich shop can sell q thousand sandwiches per week, then the equations $f(4.5) = 1.2$, $f'(4.5) = -0.1$ mean

(A) When the price of a sandwich is \$4.50, the shop can sell 1200 sanwiches per week and the profit is decreasing at \$100 per week.

(B) The graph of f is a straight line going through (4.5 , 1.2) with slope -1.

(C) When the price of a sandwich is \$1.20, the shop can sell 4500 sandwiches per week, but the shop loses \$100 per week.

(D) When the price of a sandwich is \$4.50, the shop can sell 1200 sandwiches per week, but the price is decreasing 10 cents per sandwich.

(E) None of the above.

---

23. If for a twice-differentiable function, $f(x) \cdot f'(x) \cdot f''(x) > 0$, then which of the following is possible?

(A) The graph has values only in the second quadrant and the graph is decreasing and concave up.

(B) The graph has values only in the second quadrant and the graph is decreasing and concave down.

(C) The graph has values only in the first quadrant and the graph is increasing and concave down.

(D) The graph has values only in the third quadrant and the graph is increasing and concave up.

(E) The graph has values only in the fourth quadrant and the graph is decreasing and concave down.

---

24. If $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 1$, then which of the following must be true?

I. $\lim_{x \to 2} f(x) = 1$
II. f is continuous at $x = 2$
III. $f'(2) = 1$
IV. $f(2) = 1$

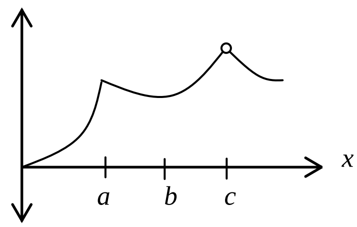
(A) I only (B) III only (C) IV only
(D) II and III only (E) I, II, and III

Part B **Graphing Calculator Required**

25. If $\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}$ does not exist, then which of the following must be true?

I. $\lim_{x\to 4} f(x)$ does not exist
II. f is not continuous at $x = 4$
III. f is not differentiable at $x = 4$
IV. $f(4)$ does not exist

(A) I only (B) III only (C) IV only
(D) II and III only (E) I, II, and III



26. The graph of f is shown above. f has a "sharp corner" at $x = a$, a horizontal tangent at $x = b$, and removable discontinuity at $x = c$. Which of these statements about f is false?

(A) f is continuous but not differentiable at $x = a$.
(B) $\lim_{x\to a^-}\frac{f(x+a)-f(x)}{a} \neq \lim_{x\to a^+}\frac{f(x+a)-f(x)}{a}$.
(C) $f(a)$ is defined, but $f(c)$ is not.
(D) $f'(b)=0$.
(E) $\lim_{x\to c^-}\left(f(x)\right)\neq \lim_{x\to c^+}\left(f(x)\right)$.

27. The following table represents the population (in millions) of a country, where the population, P, is a function of year, t, such that $P = f(t)$.

Year	1960	1970	1980	1990
Population (millions)	4.1	5.9	7.8	11.1

The population in 1994 is about

(A) 12.42 million (B) 11.32 million (C) 11.92 million
(D) 13.18 million (E) 12.78 million

Part B **Graphing Calculator Required**

28. $\lim_{x \to -2} \frac{x^2 - 4}{2x^2 - x - 10} =$

(A) $\frac{1}{9}$ (B) $\frac{1}{4}$ (C) $\frac{4}{9}$ (D) $\frac{3}{4}$ (E) nonexistence

29. $\lim_{x \to 3} \frac{x^3 - 27}{\sqrt{x} - \sqrt{3}} =$

(A) $3\sqrt{3}$ (B) $12\sqrt{3}$ (C) $27\sqrt{3}$ (D) $54\sqrt{3}$ (E) nonexistence

Also study the Ch 1 AP Review Worksheet!!!

ANSWERS:							
1 B	5 E	9 E	13 B	17 C	21 A	25 B	29 D
2 E	6 C	10 B	14 B	18 D	22 E	26 E	
3 C	7 D	11 A	15 D	19 E	23 B	27 A	
4 A	8 E	12 D	16 E	20 B	24 D	28 C	