AP Calculus Name:____________________

The Second Fundamental Theorem of Calculus

If f is a continuous function on an interval, and if c is a constant in that interval, such that the upper limit of the integral is x and the lower limit is the constant c, then

$$\frac{d}{dx}\left(\int_c^x f(t)\ dt\right) = f(x).$$

In short, since the derivative and integral are really opposite operations, they cancel leaving us $f(x)$.

Thus, applying the chain rule,

$$\frac{d}{dx}\left(\int_c^{u(x)} f(t)\ dt\right) = f(u(x)) \cdot u'(x).$$

In other words, just plug in $u(x)$ into the t in $f(t)$, then apply the chain rule and multiply by $u'(x)$.

1. $\frac{d}{dx}\left(\int_1^x \cos t\ dt\right) =$

2. $\frac{d}{dx}\left(\int_e^x \cos t\ dt\right) =$

3. $\frac{d}{dx}\left(\int_x^1 \ln t\ dt\right) =$

4. $\frac{d}{dx}\left(\int_2^{x^2} e^t\ dt\right) =$

5. $\frac{d}{dx}\left(\int_\pi^{\sin x} \sqrt{t}\ dt\right) =$

6. $\frac{d}{dx}\left(\int_{x^2}^{\tan x} t^2\ dt\right) =$

7. $\frac{d}{dx}\left(\int_1^{x^2} \cos(\sqrt{t})\, dt\right) =$

(A) $-2x\cos\sqrt{x}$

(B) $-\cos\sqrt{x}$

(C) $2x\cos x$

(D) $-2x\cos x$

(E) $\cos\sqrt{x}$

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8. If $g(x) = \int_{x^2}^{e} \frac{1}{3t^2+1}\, dt$, then $g'(x) =$

(A) $\frac{-1}{3x^2+1}$

(B) $\frac{2x}{3x^2+1}$

(C) $\frac{-1}{3x^4+1}$

(D) $\frac{-2x}{3x^4+1}$

(E) $\frac{-12x^3}{3x^4+1}$

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9. If $f(x) = \int_1^{x} \frac{1}{t^2-4}\, dt$, then the domain of f is

(A) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

(B) $(0, 2) \cup (2, \infty)$

(C) $[1, 2) \cup (2, \infty)$

(D) $(-2, 2)$

(E) $[1, 2)$

10. If $g(x)=\int_x^4 e^{\sqrt{t}}\,dt$, then $g'(4)=$

(A) 0

(B) e^2

(C) $-e^2$

(D) -1

(E) $\dfrac{e^2}{4}$

CALCULATOR REQUIRED

11. If $g(x)=\int_1^x \cos(t^2)\,dt$ for $-1\le x\le e$, then $g(x)$ decreases on the interval

(A) $-1\le x\le 2.171$

(B) $2.171\le x\le e$

(C) $-1\le x\le 1.253$

(D) $0\le x\le 1.253$

(E) $1.253\le x\le 2.171$

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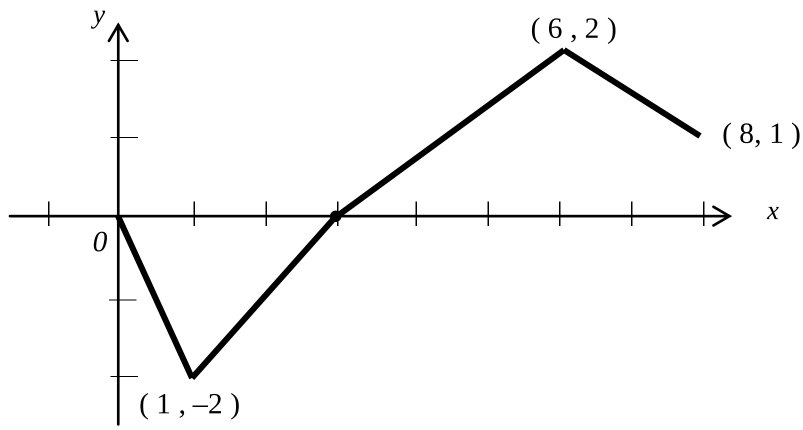
12. Let $F(x)=\int_2^{x^2} 24+29\sqrt{t}-4t\;dt$. Classify each critical point of F as a relative maximum, minimum, or neither.



Graph of f

13. Let $g(x)=\int_0^x f(t)\ dt$. Using the graph of $f(t)$ above, then find

a) $g(0)$ b) $g(1)$ c) $g'(1)$

d) The interval where g is concave up.

e) The value of x where g has a relative extrema on the interval $0 \le x \le 8$.

f) The value of x where g takes its (absolute) maximum of the interval $0 \le x \le 8$.

ANSWERS:

1) $\cos x$	7C	12) max at-3/4,8
2) $\cos x$	8D	min at 0
3) $-\ln x$	9D	13 a) 0
4) $2xe^x$	10C	b) -1
5) $\cos t\sqrt{\sin x}$	11E	c) -2
6) $-2x^5+\tan^2 x\sec^2 x$		d) 1 < x < 6
		e) x = 8