1. Use the exponential formula for hyperbolic functions to simplify the expressions.

a) $\sinh(\ln t^2)$

$\frac{1}{2}e^{\ln t^2} - \frac{1}{2}e^{-\ln t^2}$

$\frac{1}{2}t^2 - \frac{1}{2}(e^{\ln t^2})^{-1}$

$\frac{1}{2}t^2 - \frac{1}{2}(t^2)^{-1}$

$\boxed{\frac{1}{2}t^2 - \frac{1}{2t^2}}$

b) $\tanh(2+\ln t)$

$\frac{e^{2+\ln t} - e^{-(2+\ln t)}}{e^{2+\ln t} + e^{-(2+\ln t)}}$

$\frac{e^2 e^{\ln t} - (e^2 e^{\ln t})^{-1}}{e^2 e^{\ln t} + (e^2 e^{\ln t})^{-1}}$

$\frac{e^2 t - \frac{1}{e^2 t}}{e^2 t + \frac{1}{e^2 t}} \frac{(e^2 t)}{(e^2 t)}$

$\boxed{\frac{e^4 t^2 - 1}{e^4 t^2 + 1}}$

2. Use the exponential formula for hyperbolic functions to show that the properties to evaluate each limit.

a) $\lim_{x\to\infty} \frac{\cosh(2x)}{\sinh(3x)}$

$\lim_{x\to\infty} \frac{\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}}{\frac{1}{2}e^{3x} - \frac{1}{2}e^{-3x}}$

$\lim_{x\to\infty} \frac{\frac{1}{2}e^{2x} + \frac{1}{2}\frac{1}{e^{2x}}}{\frac{1}{2}e^{3x} - \frac{1}{2}\frac{1}{e^{3x}}}$

$\lim_{x\to\infty} \frac{\frac{1}{2}e^{2x} + 0}{\frac{1}{2}e^{3x} + 0} = \lim_{x\to\infty} \frac{1}{e^x} = \boxed{0}$

b) $\lim_{x\to\infty} \frac{e^{2x}}{\sinh(2x)}$

$\lim_{x\to\infty} \frac{e^{2x}}{\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x}}$

$\lim_{x\to\infty} \frac{e^{2x}}{\frac{1}{2}e^{2x} - \frac{1}{2}\frac{1}{e^{2x}}}$

$\lim_{x\to\infty} \frac{e^{2x}}{\frac{1}{2}e^{2x} - 0}$

$= \frac{1}{\frac{1}{2}} = \boxed{2}$

3. Find the derivatives.

a) $y = \sinh(2x)$

$\boxed{y' = 2\cosh 2x}$

b) $y = \cosh^2 t + \cosh(e^{t^2})$

$y' = 2\cosh t \sinh t + (\sinh e^{t^2})\, e^{t^2} \cdot 2t$

$\boxed{y' = 2\cosh t \sinh t + 2te^{t^2}\sinh(e^{t^2})}$

c) $y = \dfrac{\cosh t^3}{\sinh t}$

$$y' = \boxed{\frac{(\sinh t^3)(3t^2)\sinh t - \cosh t^3 \cosh t}{(\sinh t)^2}}$$

4. Consider the function $y = \sinh(2 - \sqrt{x})$ for $x > 0$.

a) Find the equation of the tangent line to the graph of y at $x = 4$.

$(4, 0)$

$x = 4$ $\quad y = \sinh(2 - \sqrt{4})$

$y = \sinh 0$

$y = 0$

$m = [\cosh(2 - \sqrt{x})](-\frac{1}{2}x^{-\frac{1}{2}}) \Big|_{x=4}$

$m = [\cosh(2 - \sqrt{4})](-\frac{1}{2}\frac{1}{\sqrt{4}})$

$m = (\cosh 0)(-\frac{1}{2}\cdot\frac{1}{2})$

$m = 1 \cdot (-\frac{1}{4})$

$m = -\frac{1}{4}$

$\therefore \boxed{y = -\frac{1}{4}(x - 4)}$

b) Use the equation of the tangent line at $x = 4$ to estimate $y(4.1)$.

$y(4.1) = -\frac{1}{4}(4.1 - 4) = \boxed{-.025}$

c) Determine if the estimate $y(4.1)$ is an overestimate or underestimate. Justify your answer.

$y' = [\cosh(2 - \sqrt{x})](-\frac{1}{2}x^{-\frac{1}{2}})$

$y'' = [\sinh(2 - \sqrt{x})](-\frac{1}{2}x^{-\frac{1}{2}})(-\frac{1}{2}x^{-\frac{1}{2}}) + [\cosh(2 - \sqrt{x})][\frac{1}{4}x^{-\frac{3}{2}}]$

$y''(4) = \sinh(2 - \sqrt{4})(-\frac{1}{2}\frac{1}{\sqrt{4}})(-\frac{1}{2}\frac{1}{\sqrt{4}}) + [\cosh(2 - \sqrt{4})][\frac{1}{4}(4)^{-\frac{3}{2}}]$

$y''(4) = \frac{1}{4}(4)^{-\frac{3}{2}} > 0$

$= \frac{1}{4}(\frac{1}{8}) = \frac{1}{32} > 0$

$\therefore$ $y(4.1)$ is underestimate since $y''(4) > 0 \Rightarrow y$ concaves up. The tangent line lies below curve.

Starting with $x_0 = 1$, use TWO iterations of Newton's Method to determine an approximation the solution of the following equation.

5. $x^2 + \ln x = 2$

$x_0 = 1$

$x^2 + \ln x - 2 = 0$

$y = x^2 + \ln x - 2$

$y' = 2x + \frac{1}{x}$

$$x_1 = 1 - \frac{(1)^2 + \ln(1) - 2}{2(1) + \frac{1}{(1)}} = 1 - \frac{1}{3} = \frac{4}{3}$$

$$x_2 = \frac{4}{3} - \frac{(\frac{4}{3})^2 + \ln(\frac{4}{3}) - 2}{2(\frac{4}{3}) + \frac{1}{(\frac{4}{3})}} = \frac{4}{3} - \frac{.0654545}{3.416} = \boxed{1.314}$$

Use the Binomial Expansion Theorem to expand the expressions.

6. $(3x - 4)^6$

$$1(3x)^6(4)^0 - 6(3x)^5(4)^1 + 15(3x)^4(4)^2 - 20(3x)^3(4)^3 + 15(3x)^2(4)^4 - 6(3x)^1(4)^5 + 1(3x)^0(4)^6$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$\boxed{729x^6 - 5832x^5 + 19440x^4 - 34{,}560x^3 + 34{,}560x^2 - 18{,}432x + 4096}$$

Evaluate the limits algebraically. You may not use L'Hôpital's Rule.

7. $\lim\limits_{x \to 0} \frac{(8)x}{\sin 8x}\left(\frac{1}{8}\right)$

$= \boxed{\frac{1}{8}}$

8. $\lim\limits_{x \to 0} \frac{\cos^2 4x}{x^2 \cot^2 3x}$

$$= \lim_{x \to 0} (3)(3)\left(\frac{\cos 4x}{1}\right)\left(\frac{\cos 4x}{1}\right)\left(\frac{\sin 3x}{(3)x}\right)\left(\frac{\sin 3x}{(3)x}\right)\frac{1}{(\cos 3x)^2}$$

$3 \cdot 3 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{1}{1^2}$

$\boxed{9}$

9. $\lim_{x\to 0}\frac{5x^2}{\cos 2x-1}$ $\frac{(\cos 2x+1)}{(\cos 2x+1)}$

$\lim_{x\to 0}\frac{5x^2(\cos 2x+1)}{\cos^2 2x-1}$

$\cos^2\theta+\sin^2\theta=1$

$\cos^2\theta-1=-\sin^2\theta$

$\lim_{x\to 0}\frac{5x^2(\cos 2x+1)}{-\sin^2 2x}$

$\lim_{x\to 0}-5\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{2x}{\sin 2x}\cdot\frac{2x}{\sin 2x}(\cos 2x+1)$

$=5\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 1\cdot 1\cdot(1+1)=\boxed{\frac{5}{2}}$

10. $\lim_{x\to 0}\frac{x^2}{\sec x-1}$

$\lim_{x\to 0}\frac{x^2}{\frac{1}{\cos x}-1}$ $\frac{\cos x}{\cos x}$

$\lim_{x\to 0}\frac{x^2\cos x}{1-\cos x}$ $\frac{(1+\cos x)}{(1+\cos x)}$

$\lim_{x\to 0}\frac{x^2\cos x(1+\cos x)}{1-\cos^2 x}$

$\lim_{x\to 0}\frac{x^2\cos x(1+\cos x)}{\sin^2 x}$

$\lim_{x\to 0}\frac{x}{\sin x}\frac{x}{\sin x}\cos x(1+\cos x)$

$1\cdot 1\cdot 1\cdot(1+1)$

$=\boxed{2}$

11. $\lim_{x\to\frac{\pi}{3}}\frac{\frac{1}{2}-\cos x}{x-\frac{\pi}{3}}$ [Hint: $\cos(a+b)=(\cos a)(\cos b)-(\sin a)(\sin b)$]

Let $A=x-\frac{\pi}{3}$

$x=A+\frac{\pi}{3}$

$\lim_{A\to 0}\frac{\frac{1}{2}-\cos(A+\frac{\pi}{3})}{A}$

$\lim_{A\to 0}\frac{\frac{1}{2}-\cos A(\cos\frac{\pi}{3})+\sin A(\sin\frac{\pi}{3})}{A}$

$\lim_{A\to 0}\frac{\frac{1}{2}-\cos A(\frac{1}{2})+\sin A(\frac{\sqrt{3}}{2})}{A}$

$=\frac{\frac{1}{2}-\cos 0(\frac{1}{2})+\sin A(\frac{\sqrt{3}}{2})}{A}=\frac{\frac{1}{2}-1(\frac{1}{2})\sin A(\frac{\sqrt{3}}{2})}{A}$

$=\lim_{A\to 0}\frac{\sin A(\frac{\sqrt{3}}{2})}{A}$

$1(\frac{\sqrt{3}}{2})$

$\boxed{\frac{\sqrt{3}}{2}}$

For each pair of parametric equations in problems #12 and #13, (a) fill in the table, (b) write the equation in terms of x and y, and (c) sketch the graph.

12. $x(t)=2t+1$ and $y(t)=t^2$

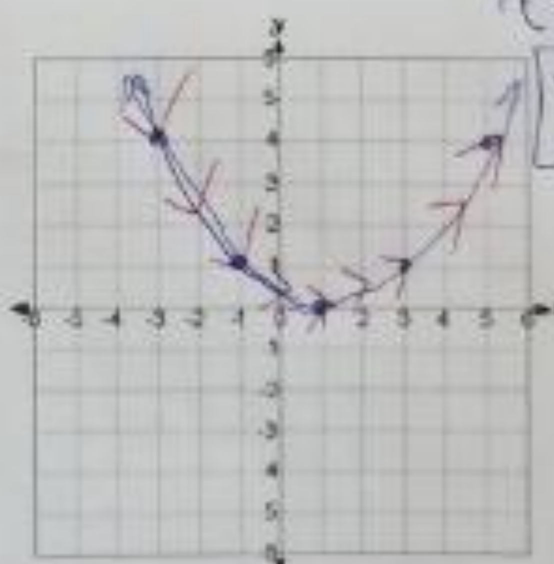
$2t=x-1$

$t=\frac{x-1}{2}$

$\boxed{y=\left(\frac{x-1}{2}\right)^2}$

OR $y=\frac{1}{4}(x-1)^2$

t	x	y
-2	-3	4
-1	-1	1
0	1	0
1	3	1
2	5	4



13. $x(t) = 4\cos t$ and $y(t) = 4\sin t$

$\cos t = \frac{x}{4}$ $\sin t = \frac{y}{4}$

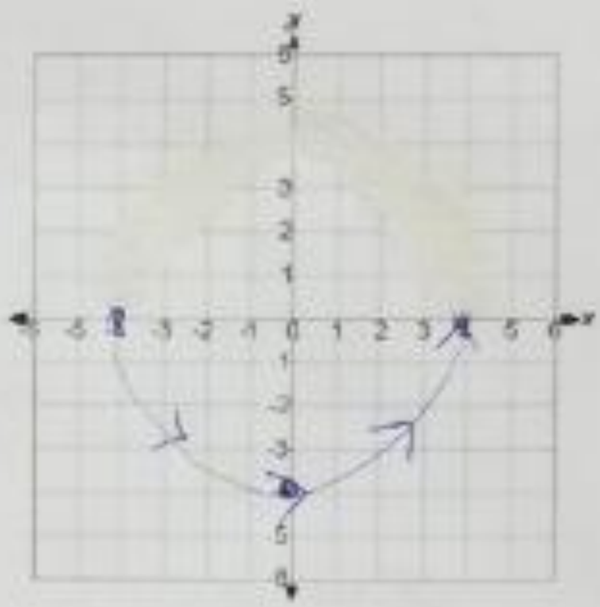
t	x	y
$-\pi$	-4	0
$-\frac{\pi}{2}$	0	-4
0	0	4
$\frac{\pi}{2}$	-4	0
π	0	-4

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

$\frac{x^2}{16} + \frac{y^2}{16} = 1$

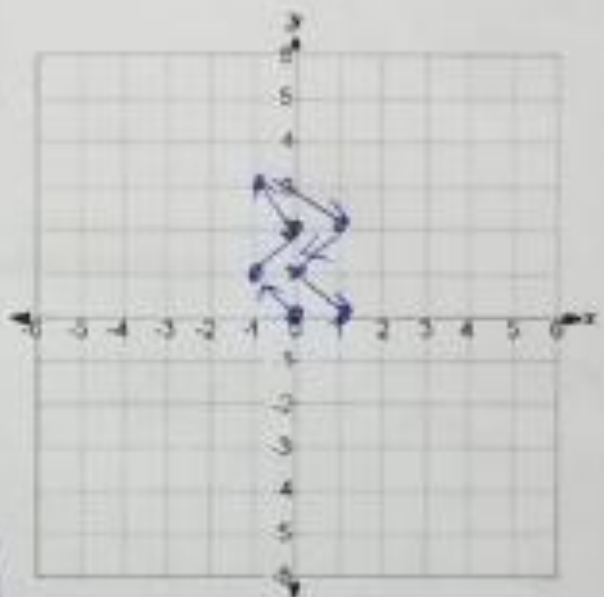
$\boxed{x^2 + y^2 = 16}$



14. Given the graphs below, draw the graph of the parametric equations $x = a(t)$, $y = b(t)$.

t	x	y
0	0	0
1	-1	1
2	0	2

t	x	x
3	-1	3
4	1	2
5	0	1
6	1	0



15. Write parametric equations of a line passing through the points (3 , -1) and (-2 , -3).

$t=0$ $t=1$

$$\boxed{\begin{aligned} x &= -5t + 3 \\ y &= -2t - 1 \end{aligned}}$$

16. Write parametric equations of the circle $x^2 + y^2 = 25$. $\leftarrow r = 5$

$x = r\cos\theta \quad y = r\sin\theta$

$$\boxed{x = 5\cos\theta \quad y = 5\sin\theta}$$

17. Write parametric equations of the ellipse $\frac{x^2}{81} + \frac{y^2}{100} = 1$.

$a^2 = 9 \quad b^2 = 100$

$a = 3 \quad b = 10$

$$\boxed{\begin{aligned} x &= 3\cos\theta \\ y &= 10\sin\theta \end{aligned}}$$

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18. Find the polar coordinates of the point with the given rectangular coordinates.

a. $(-8, 15)$

$x^2 + y^2 = r^2$

$(-8)^2 + (15)^2 = r^2$

$289 = r^2$

$17 = r$

$\theta = \tan^{-1}\left(\frac{15}{-8}\right)$

$\theta = -1.0808$ radians

or

$\theta = -61.9275°$

15

$\boxed{(-17, -61.9275°)}$

OR -8

$\boxed{(17, 118.0725°)}$

b. $(-2\sqrt{3}, -2)$

$x^2 + y^2 = r^2$

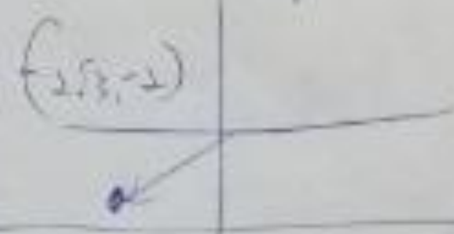
$(-2\sqrt{3})^2 + (-2)^2 = r^2$

$r^2 = 16$

$r = 4$

$\theta = \tan^{-1}\left(\frac{-2}{-2\sqrt{3}}\right)$

$\theta = \frac{\pi}{6}$



$$\boxed{\left(-4, \frac{\pi}{6}\right) \text{ or } \left(4, \frac{7\pi}{6}\right)}$$

19. Find the rectangular coordinates of the point with polar coordinates

a. $\left(3, -\frac{4\pi}{3}\right)$

$x = r\cos\theta \quad y = r\sin\theta$

$x = 3\cos\left(\frac{-4\pi}{3}\right) \quad y = 3\sin\left(\frac{-4\pi}{3}\right)$

$x = 3\left(-\frac{1}{2}\right) \quad y = 3\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)}$

b. $\left(5, \frac{\pi}{4}\right)$

$x = 5\cos\frac{\pi}{4} \quad y = 5\sin\frac{\pi}{4}$

$x = 5\left(\frac{\sqrt{2}}{2}\right) \quad y = 5\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)}$

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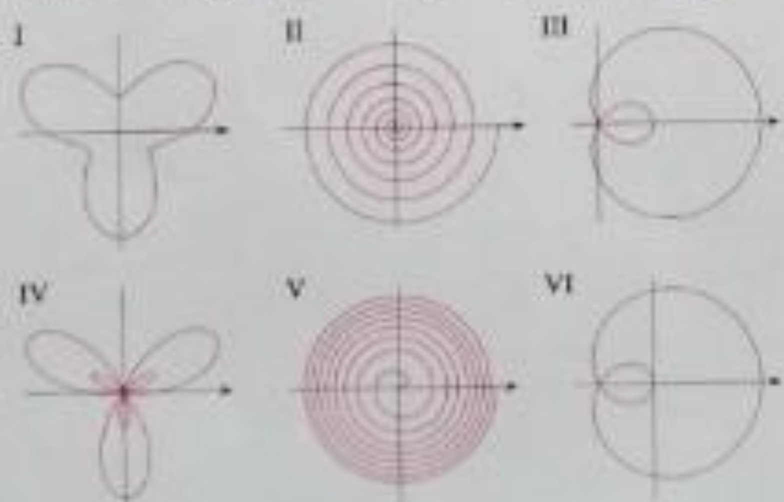
20. Match the polar equations with the graphs labeled I–VI. Give reasons for your choices. (Don't use a graphing device.)

(a) $r = \sqrt{\theta}$ II $0 \le \theta \le 16\pi$ (b) $r = \theta^2$, V $0 \le \theta \le 16\pi$

(c) $r = \cos(\theta/3)$ VI (d) $r = 1 + 2\cos\theta$ III

(e) $r = 2 + \sin 3\theta$ I (f) $r = 1 + 2\sin 3\theta$ IV



This is JFF. Won't be on Test!

21. Write the rectangular equations in polar form.

a. $y = 3x$

$\frac{y}{x} = 3$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\boxed{\theta = \tan^{-1} 3}$

b. $x^2 + y^2 = 25$

$r^2 = 25$

$\boxed{r = 5}$

22. Write the polar equations in rectangular form.

a. $r = 2\csc\theta$

$r = \frac{2}{\sin\theta}$

$r\sin\theta = 2$

$\boxed{y = 2}$

b. $r = -4\sin\theta$ $\cdot r$

$r^2 = -4 r\sin\theta$

$x^2 + y^2 = -4y$

$x^2 + y^2 + 4y = 0$

$x^2 + (y^2 + 4y + 4) = 0 + 4$

$\boxed{x^2 + (y+2)^2 = 4}$