A particle moves along a coordinate line, where its position at time $t$ seconds, for $t \geq 0$, is given by $x(t) = -t^2 + 4t + 5$ feet. Note that the vertex of the parabola is at $t = 2$. Find the position, velocity, speed, and acceleration at time $t = 2.5$, and the total distance traveled from time $t = 0$ to $t = 2.5$.

1. position: $x(2.5) = - (2.5)^2 + 4(2.5) + 5 = 8.75$
2. velocity: $v(t) = x'(t) = -2t + 4$. Thus, $v(2.5) = -1$
3. speed: $|v(2.5)| = |-1| = 1$
4. acceleration: $a(t) = v'(t) = -2$
5. distance traveled from $t = 0$ to $t = 2.5$:

6. Find the total distance traveled time $t = 0$ to $t = 5$ seconds.

7. A particle moves along the x-axis so that at time $t \geq 0$, its position is given by $x(t) = \frac{4}{3}t^3 - 14t^2 + 49t - 53$. At what time $t$ is the particle at rest?

(A) $t = 1$ only
(B) $t = 3$ only
(C) $t = \frac{7}{2}$ only
(D) $t = 3$ and $t = \frac{7}{2}$
(E) $t = 3$ and $t = 4$

8. A particle moves along the x-axis so that at time $t \geq 0$, its velocity is given by $v(t) = 5 - 4.9 \text{sec}^2(5t)$. What is the acceleration of the particle at time $t = 3$?

(A) -32.016
(B) -0.677
(C) 19.053
(D) 44.185
(E) 72.682

9. A particle moves along a horizontal path so that at time $t \geq 0$, its velocity is given by $v(t) = \sin(0.15t^3 + 1)$. How many times does the particle change directions on the interval $0 \leq t \leq 5$?

(A) Two  (B) Four  (C) Five  (D) Six  (E) None

Count xprints that change sign.
10. A particle moves along a sky with the velocity \( v(t) = e^t \sin t \). How many relative extrema does the particle experience during the first ten seconds?

(A) One  
(B) Two  
(C) Three  
(D) Four  
(E) Five

11. The position function, \( x(t) = -4.905t^2 + v_0t + x_0 \), is the height of the object, measured in meters, at time \( t \) seconds. If an explosion on the top of a 50-meter tower causes debris to rise vertically with an initial 72 m/s, then

(a) Find the velocity of the object at time \( t = 3 \).
(b) Find its maximum height.
(c) Find the velocity when the object is at a height of 32 meters.

\[
x(t) = -4.905t^2 + 72t + 50
\]

\[
\begin{align*}
v(t) &= x'(t) = -9.81t + 72 \\
v(3) &= -9.81(3) + 72 = 41.257 \text{ m/s}
\end{align*}
\]

\[
\text{Max h_t} = 314.220 \text{ m}
\]

\[
\begin{align*}
\text{Find } v(t) \text{ when } x(t) &= 32 \\
-4.905t^2 + 72t + 50 &= 32 \\
-4.905t^2 + 72t + 18 &= 0 \\
t &= \frac{-72 \pm \sqrt{72^2 - 4(-4.905)(18)}}{-9.81} \\
t &= \frac{-72 \pm 73.244}{-9.81} \\
t &\approx -7.4588, 14.9247
\end{align*}
\]

\[
v(14.9247) = -74.412 \text{ m/s}
\]
12. A particle moves along the y-axis so that its velocity $v$ at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reached its highest point. Justify your answer.

\[ a(t) = v'(t) = -\frac{1}{1+(e^t)^2} \cdot e^t \]

\[ a(2) = \frac{-e^2}{1+e^4} \approx -1.329 \]

(b) If $v(t)$ and $a(t)$ have the same signs, then speed increases.
   If $v(t)$ and $a(t)$ have different signs, then speed decreases.

\[ a(2) < 0 \quad v(2) = 1 - \tan^{-1}(e^2) = -0.43867 < 0 \]

\[ \Rightarrow \text{Speed is increasing at } t = 2 \text{ since } a(2) < 0 \text{ and } v(2) < 0. \]

Verify graphically:

(c) Find abs. max of $x(t)$.

Graphical Analysis

Particle gets up to $y = 0.443022$, after that the particle goes down.

$x'(t) = 0, \quad v(t) = 0, \quad t = 0.443022$; this is a rel. max since $v$ changes from pos to neg.

Since this is the only rel. extrema, the particle must be an abs. max at $t = 0.443022$. 