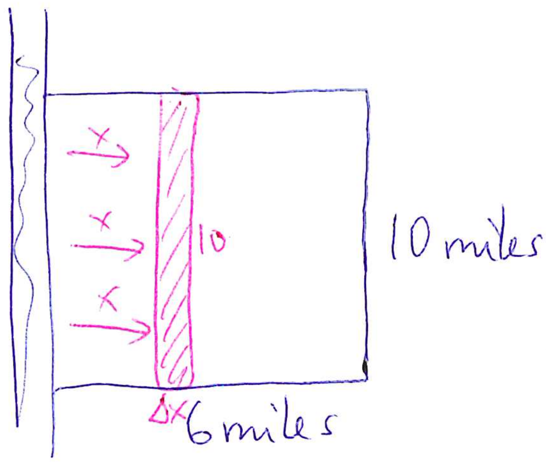
Population Density Wkst

Name: Key

A Graphing Calculator is allowed for these problems.

1. A rectangular shaped city lays along side a straight river. Its population decreases as you move away from the river. The population can be estimated as $20 - 3x$ thousand per square mile.

   a) If the city is 10 miles in length along the river and 6 miles wide, then find its population.



$$\text{Pop of slice} = (20-3x)\,10\,\Delta x$$

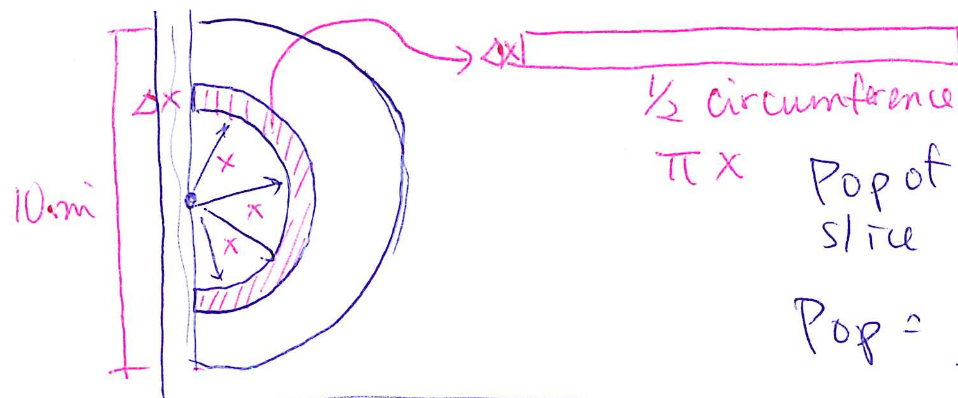
$$\text{Pop} = \int_0^6 (20-3x)\,10\,dx$$

$$= 660 \text{ thousands}$$

$$= \boxed{660{,}000}$$

   b) Suppose the city is semicircular in shape with a diameter of 10 miles. Find its population if

      i) the density varies as you move away from the center point along the river.



$$\text{Pop of slice} = (20-3x)\,\pi x\,\Delta x$$

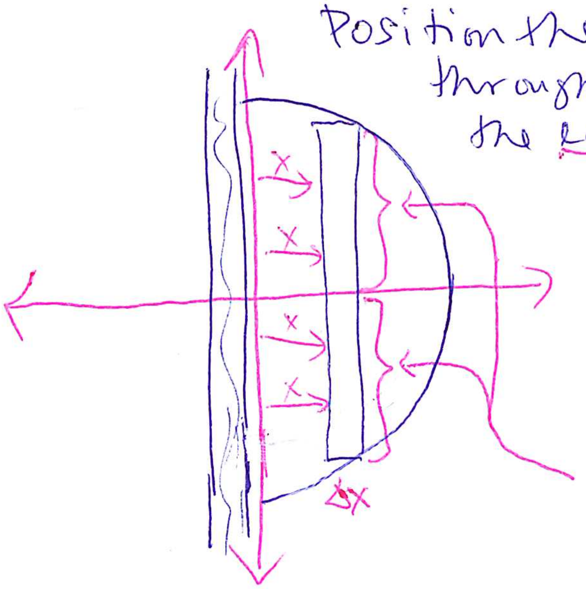
$$\text{Pop} = \int_0^5 (20-3x)\,\pi x\,dx$$

$$= 125\pi \text{ thousands}$$

$$= \boxed{392{,}699}$$

      ii) the density varies as you move away from the river.

Position the xy-axis through the city & find the equation of the circle.

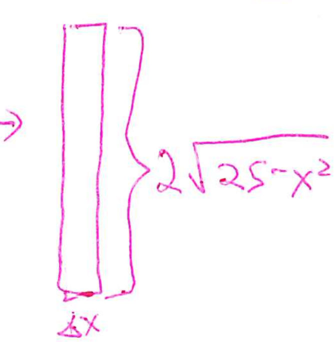


$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 5^2$$

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}$$



$$\text{Pop of slice} = (20-3x)\,2\sqrt{25-x^2}\,\Delta x$$

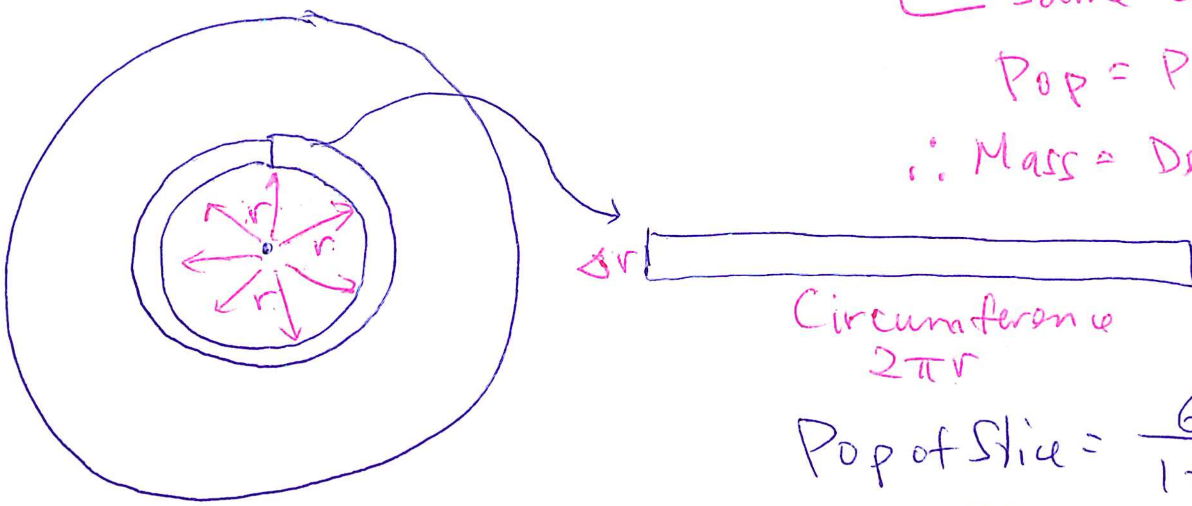
$$\text{Pop} = \int_0^5 (20-3x)\,2\sqrt{25-x^2}\,dx$$

$$= \boxed{535{,}398}$$

2. A circular oil spill has a density at r meters from its center given by $\dfrac{60}{1+r^2}$ kg / m².

a) If the oil spill extends to 2000 meters in radius, then find its mass.

Same Concept as
Pop = Popdensity · Area
∴ Mass = Density · Area



$$\text{Pop of Slice} = \frac{60}{1+r^2} \cdot 2\pi r \Delta r$$

$$\text{Pop} = \int_0^{2000} \frac{60}{1+r^2} \cdot 2\pi r \, dr$$

$$= \boxed{2865.473 \text{ kg}}$$

b) Within what distance is half the oil slick contained?

ie) Find the radius where the total mass is ½(2865.473). You will need to find the upper limit of the integral.

$$\int_0^x \frac{60}{1+r^2} \cdot 2\pi r \, dr = \frac{1}{2}(2865.473)$$

$$\int_0^x \frac{120\pi r}{1+r^2} dr = 1432.736383$$

$$120\pi \int_0^x \frac{r}{1+r^2} dr = 1432.736383 \longrightarrow 120\pi\left[\frac{1}{2}\ln(1+x^2)\right] = 1432.73638$$

Let $u = 1 + r^2$
$du = 2r\,dr$
$\frac{1}{2}du = r\,dr$

$$\frac{1}{2}\int_1^{1+x^2} \frac{1}{u} du$$

$$\frac{1}{2}\ln|u|\Big|_1^{1+x^2}$$

$$\frac{1}{2}\ln(1+x^2) - \underbrace{\frac{1}{2}\ln 1}_{0}$$

$$\frac{1}{2}\ln(1+x^2)$$

$$\frac{60\pi \ln(1+x^2)}{60\pi} = \frac{1432.736383}{60\pi}$$

$$\ln(1+x^2) = 7.600902585$$

$$1 + x^2 = e^{7.600902585}$$

$$x = \sqrt{e^{7.600902585} - 1}$$

$$x = \boxed{44.71018061 \text{ m}}$$

Answers:			
1a. 660,000	1bi. 392,699	2a. 2865.473 kg	2b. 44.710 m
bii. 535,398			