1. A rectangular shaped city lays along side a straight river. Its population decreases as you move away from the river. The population can be estimated as $20 - 3x$ thousand per square mile.
   a) If the city is 10 miles in length along the river and 6 miles wide, then find its population.

   

   $\text{Pop of slice} = (20 - 3x) \times 10 \, \text{dx}$

   $\text{Pop} = \int_0^6 (20 - 3x) \times 10 \, dx$

   $= 660 \text{ thousands}$

   $= 660,000$

   b) Suppose the city is semicircular in shape with a diameter of 10 miles. Find its population if
      i) the density varies as you move away from the center point along the river.

   

   $\frac{1}{2} \text{ circumference} = \pi \times x$

   $\text{Pop of slice} = (20 - 3x) \times \pi \times dx$

   $\text{Pop} = \int_0^5 (20 - 3x) \times \pi \, dx$

   $= 125 \pi \text{ thousand}$

   $= 392,699$

   ii) the density varies as you move away from the river.

   Position the $xy$-axis through the city and find the equation of the circle:

   $x^2 + y^2 = 25$

   $x + y = 5$

   $y = 25 - x^2$

   $y = \pm \sqrt{25-x^2}$

   $\text{Pop of slice} = (20 - 3x) \times 2\sqrt{25-x^2} \, dx$

   $\text{Pop} = \int_{-5}^{5} (20 - 3x) \times 2\sqrt{25-x^2} \, dx$
2. A circular oil spill has a density at \( r \) meters from its center given by \( \frac{60}{1+r^2} \) kg/m².

   a) If the oil spill extends to 2000 meters in radius, then find its mass.

   \[
   \text{Pop of Slice} = \frac{60}{1+r^2} \cdot 2\pi r \, dr
   \]

   \[
   \text{Pop} = \int_0^{2000} \frac{60}{1+r^2} \cdot 2\pi r \, dr
   \]

   \[= \frac{2865.473}{1}
   \]

   b) Within what distance is half the oil slick contained?

   i.e. Find the radius where the total mass is \( \frac{1}{2} \left( \frac{2865.473}{1} \right) \).

   You will need to find the upper limit of the integral.

   \[
   \int_0^x \frac{60}{1+r^2} \cdot 2\pi r \, dr = \frac{1}{2} \left( \frac{2865.473}{1} \right)
   \]

   \[
   120\pi \left[ \ln(1+x^2) \right] = 1432.736383
   \]

   Let \( u = 1 + r^2 \)

   \[
   du = 2r \, dr
   \]

   \[
   \frac{1}{2} \ln(u) \bigg|_{1}^{1+x^2} = \frac{1}{2} \ln(1+x^2) - \frac{1}{2} \ln(1)
   \]

   \[
   x = \sqrt{\frac{7.60090255 - 1}{1}} = 44.718861 \text{ m}
   \]