During the fall, Ben’s family operates a corn maze on their farm. They charge $8 per visitor. The total amount of money they collect is the number of visitors times $8. If \( n \) represents the number of visitors, then the total amount of money collected is \( 8n \).

A variable is a letter or symbol that represents a quantity that can change. \( 8n \) is an expression that represents a quantity. In this situation, \( 8n \) is the total amount of money collected.

You can use variables and expressions to solve problems.

**Problem 2.1**

A. The family wanted an estimate of how much money they might receive from maze visitors on any given day. They decided to make a table that would display this amount for several different numbers of visitors for one day.

1. Copy and complete the table for numbers of visitors in increments of 5, starting with 0 and ending with 100.

<table>
<thead>
<tr>
<th>Number of Visitors, ( n )</th>
<th>Amount of Money Collected, ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

2. Sketch a graph of the number of visitors and the total amount of money collected.

3. Describe the shape of the graph.

B. Ben estimates that the cost of maintaining and advertising the maze is $75 per day.

1. Write an expression that represents the amount of profit that the family expects to make each day the maze is open.

2. Calculate the profit for 80 visitors, 105 visitors, and 120 visitors.

C. 1. Add a third column to the table in Part A that represents the profit, \( p \).

2. Graph the number of visitors and profit on the same graph as in Part A.

3. Compare the shape of this graph to the one in Part A.
The total amount of money collected also is a variable. It depends on the number of visitors for any given day. We call the total amount of money collected the dependent variable and the number of visitors the independent variable.

**Problem 2.2**

The corn maze is open on weekends. Let \( n \) represent the number of visitors on Saturday, and let \( m \) represent the number of visitors on Sunday.

**A.**

1. Write an expression that represents the total amount of money collected for both days.
2. Ben claims there is more than one way to write this expression. Do you agree? Explain.
3. What is the total amount of money collected for the weekend if there were 75 visitors on Saturday and 90 visitors on Sunday?

**B.**

1. Write an expression for the total profit for both days if the expenses are $75 per day.
2. Is there a profit for a total of 150 visitors on both days? Explain.
3. What is the least number of visitors needed on a weekend to break even? This is when the total revenue, the money taken in, equals the total expenses. Explain how you found your answer.

**Problem 2.3**

The family charges $3 per person for tractor rides around the farm.

**A.** Write an expression that represents the total amount of money collected in one day from visitors to the maze and tractor rides. Explain what your variables represent.

**B.** If the combined expenses for the maze and tractor rides are $160 per day, write an expression for the profit for one day. Explain what your variables represent.

**C.** Calculate the profit for each.

1. 70 maze visitors and 40 tractor rides
2. 90 maze visitors and 65 tractor rides
Ben and his older sister, Emma, help out on their family’s farm by grooming horses and mowing the fields.

A. It takes Ben 30 minutes to groom each horse on the farm.
   1. Write an algebraic expression that shows how long it takes Ben to groom some horses. Explain what the variable in your expression represents.
   2. How long does it take Ben to groom 5 horses?
   3. Ben spends $t$ minutes grooming horses before lunch. Write an algebraic expression to show how much time Ben will need after lunch to finish grooming the 5 horses.
   4. What information do you need to be able to evaluate the expression?

B. It takes Ben’s older sister, Emma, 20 minutes to groom each horse on the farm.
   1. Write an expression to show how long it takes Emma to groom $h$ horses.
   2. Write an expression to show how much longer it takes Ben to groom $h$ horses than it takes Emma.
   3. Evaluate your expression for $h = 4$. Explain what the value means.

C. Three of the fields on the farm are squares with the same area.
   1. Write an expression to show the total area of the 3 fields in terms of their side length $s$.
   2. Evaluate your expression to find the total area of the fields if they each measure $\frac{1}{2}$ mile on a side.

Problem 2.4

Problem 2.5

A. It takes Ben 40 minutes to mow each field on the farm.
   1. Write an algebraic expression to represent the time that Ben spends mowing $f$ fields.
   2. Write an expression that shows how long it takes for Ben to groom $h$ horses and mow $f$ fields.
   3. Evaluate your expression for 4 horses and 4 fields. Explain what the value means.
B. Ben’s mother tells him that he needs either to groom some horses or mow some fields before he can go to a friend’s house.

1. Write an expression that shows how much longer it will take Ben to do one chore than the other.
2. Ben’s mother gives him the option of grooming 3 horses or mowing 2 fields. Which should Ben choose? Explain your answer.
3. Suppose Ben’s option is to groom 4 horses or mow 3 fields. Which should Ben choose? Explain your answer.

Problem 2.6

A. Emma knows that her plant is growing about 2 inches each week.

1. If \( g \) represents last week’s height of the plant in inches, write an expression for the height of the plant this week.
2. Today the plant measures 16 inches in height. Set your expression equal to 16.
3. How does subtracting 2 find the height of the plant last week?
4. How tall was the plant last week?
5. What would the expression \( 2g \) mean?

B. Ben just bought a rake for $9. He forgot how much money he had when he entered the hardware store.

1. If \( m \) represents the amount of money he had before he bought the rake, write an expression that represents the amount of money he has now.
2. He counts his money and finds that he has $25 left after he bought the rake. Set your expression equal to 25.
3. Ben wants to find the value of \( m \). He does not know whether he should add or subtract 9. Determine which operation is correct and explain your decision.
4. How much money did Ben have before he bought the rake?

C. Each student pays $4 to enter the school dance.

1. If \( s \) represents the number of students attending the dance, write an expression for the amount of money collected for the dance.
2. The money collected totals $168. Set your expression equal to 168. Which operation do you need to solve for \( s \)?
3. How many students came to the dance?
D. Christopher distributes sheets of paper to the class for a project. He gives each student 5 sheets. He wants to know how many sheets of paper he distributed.

1. If \( p \) represents the total number of sheets of paper, write an expression that represents the number of students in the class.

2. There are 32 students in the class. Set your expression equal to 32.

3. How many sheets of paper did Christopher distribute?

You can use the properties of operations described below to generate equivalent expressions.

**Commutative Property:**
Changing the order does not change the sum or product.

\[
3 + 7 = 7 + 3 \quad 4 \times 5 = 5 \times 4
\]

**Associative Property:**
Changing the grouping does not change the sum or product.

\[
4 + (7 + 9) = (4 + 7) + 9 \quad (6 \times 2) \times 8 = 6 \times (2 \times 8)
\]

**Distributive Property:**
The product of a number times a sum is equal to the sum of the products of that number and each addend.

\[
5 \times (6 + 11) = (5 \times 6) + (5 \times 11)
\]

**Problem 2.7**

Ahmad and Shada’s aunt keeps some square tomato gardens on her farm. This summer, rabbits have been eating the tomatoes. Ahmed learned that marigolds keep rabbits away from tomato plants. He decides to help his aunt by planting a 1-foot border of marigolds around each tomato garden.

A. 1. Ahmad writes an expression for the perimeter of a garden as \( s + s + s + s \). Shada writes the expression \( 4s \) to represent the perimeter. Whose expression is correct? Explain how you know.

2. Write an expression to represent the perimeter of a garden after a 1-foot border of marigolds is added.
B. Three of the tomato gardens have side lengths of 6 feet, 10 feet, and 13 feet.

1. Ahmed uses the expression \( s + 2 + s + 2 + s + 2 + s + 2 \) to find the perimeter after the border of marigolds is added. Use this expression to find the perimeter of each size garden.

2. Shada uses the expression \( 4(s + 2) \) to find the perimeter after the border of marigolds is added. Use this expression to find the perimeter of each size garden.

3. What do you notice about the perimeter of each garden found using the different expressions? Explain what that tells you about the expressions.

C. Their uncle says that the outside perimeter of any garden also could be found using the expression \( 4s + 8 \). Is this expression equivalent to those written by Ahmed and Shada? Explain your reasoning using the garden with side lengths of 6 feet.

Exercises

In Exercises 1–4, write an algebraic expression or equation for each.

1. five years older than Jamal’s age
2. The area is the length of a side squared.
3. The price is $1.35 per flower plus $12.50 for the vase.
4. The cost of the meal plus the 15% tip came to $12.95.
5. Super Locks charges $3,975 to install a security system and $6.00 per month to monitor the system and respond to alerts. Fail Safe charges $995 to install and $17.95 per month. Write an equation for each company relating its total cost to the number of months.
6. Maggie lives 1,250 meters from school. Ming lives 800 meters from school. Maggie walks at an average speed of 70 meters per minute, while Ming walks at an average speed of 40 meters per minute. Write equations that show Maggie and Ming’s distances from school \( t \) minutes after they leave their homes.
7. Chris has $12 to spend on prints from his digital camera. He wants one 5-in. \( \times \) 7-in. print and some 4-in. \( \times \) 6-in. prints. Write an equation to find how many prints he can order if the price of each 5-in. \( \times \) 7-in. print is $1.40 and the 4-in. \( \times \) 6-in. prints are $.20 each.
8. Jamal has a tutoring job. He charges $15 per hour. Next month, he expects his expenses to be $30. Write an equation to find the number of hours he must work next month to make a profit of $300.
Write an algebraic expression for each situation.

9. the cost of \( x \) apples at $0.49 each
10. the number of hits a 0.306 batter gets in \( b \) times at bat
11. the number of minutes it takes to read \( p \) pages at 10 minutes per page
12. the money left on a $20 gift card after spending \( y \) dollars
13. the distance traveled over \( t \) hours at \( r \) miles per hour

Evaluate each algebraic expression for \( a = 12 \) and \( b = 3 \).

14. \( a - 2 \)  
15. \( 5a \)  
16. \( a + b \)  
17. \( \frac{3a}{2b} \)

Evaluate each algebraic expression for \( d = \frac{3}{4}, e = \frac{4}{5}, \) and \( f = \frac{1}{2} \).

18. \( d + f \)  
19. \( de \)  
20. \( f - e \)  
21. \( 4d + 2f \)

Write a situation that could describe each algebraic expression.

22. \( a + 24 \)  
23. \( 365 - d \)  
24. \( 7w \)  
25. \( \frac{m}{55} \)

26. At a craft store, each package of beads costs $3.95.
   a. Write an algebraic expression for the cost for \( p \) packages of beads.
   b. Amy gives the sales clerk $20 for \( p \) packages of beads. Write an algebraic expression to represent Amy’s change.
   c. What is the greatest number of packages that Amy can buy with $20?

For Exercises 27–34, decide which operation is needed to isolate the variable. Solve the equation.

27. \( a + 6 = 14 \)  
28. \( b - 3 = 9 \)  
29. \( 4d = 12 \)  
30. \( 7 + t = 15 \)  
31. \( \frac{x}{2} = 5 \)  
32. \( \frac{n}{9} = 6 \)  
33. \( y - 13 = 29 \)  
34. \( 11h = 132 \)

35. Greg counted 11 people who got on the bus at the last stop. Now every seat is filled. How many people were on the bus before the stop if the bus has seats for 42 people?
36. There are four dozen daisies in a vase. If every person receives three daisies until the daisies are gone, how many people will get daisies?
37. A flower garden has 18 square feet of space. A packet of seeds fills 2 square feet. How many packets of seeds are needed to fill the garden?
38. Becky wants to solve the equation \( 3x = 18 \). She says that \( 18 - 3 = 15 \), so \( x = 15 \). Explain to Becky how to find the correct answer.
For Exercises 39 and 40, write an algebraic expression.

39. seven times a number  

40. a number of objects is split into 6 equal groups

For Exercises 41–44, evaluate each expression.

41. \(12x\) for \(x = 7\)  

42. \(112 \div x\) for \(x = 7\)

43. \(2x - 3\) for \(x = 9\)  

44. \(\frac{x}{5} + 6\) for \(x = 400\)

For Exercises 45 and 46, use the information below.

Jennifer pays an $80 down payment on a violin. She will pay the rest off at $20 a week.

45. What expression can Jennifer use to represent this situation? Explain what the variable represents.

46. If the violin costs $400, how long will it take Jennifer to pay for it?

For Exercises 47–50, use this information and advertisement.

Grace and Tina are planning a canoeing trip. They are deciding whether they should rent 2 single-seat canoes or 1 two-seat canoe. Also, they will need to rent a canoe carrier.

<table>
<thead>
<tr>
<th>Two-Seat Canoes</th>
<th>Single-Seat Canoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$45 per day</td>
<td>$25 per day</td>
</tr>
<tr>
<td>$25 canoe carrier per trip</td>
<td>Free canoe carrier</td>
</tr>
</tbody>
</table>

47. What information is needed to find the cost of renting the canoes?

48. What expression can Grace and Tina use to find the cost of renting a two-seat canoe?

49. What expression can they use to find the cost of renting 2 single-seat canoes?

50. For a 4-day trip, which type of canoe should Grace and Tina rent if they want to spend the least amount of money? Explain.
For Exercises 51 and 52, use the information below.
Roses cost $20 per dozen. The delivery fee for any order is $8.

51. If \( r \) represents the number of dozen roses, write an expression to represent the cost of \( r \) dozen roses including delivery.

52. What is the total cost of having 3 dozen roses delivered?

53. Lilah buys 2 board games. Each board game costs \( g \) dollars. She has a $6 credit from a previous purchase. Write an expression to represent the amount Lilah pays for the two games.

For Exercises 54–55, use the information and table below.
The Johnson family is having a party. They need to buy paper plates, plastic forks, and plastic spoons. The table shows how each is sold.

<table>
<thead>
<tr>
<th>Item</th>
<th>Number in a Package</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Plates</td>
<td>( x )</td>
</tr>
<tr>
<td>Plastic Forks</td>
<td>( y )</td>
</tr>
<tr>
<td>Plastic Spoons</td>
<td>( z )</td>
</tr>
</tbody>
</table>

54. a. The Johnsons bought 3 packages of paper plates and 4 packages of plastic forks. Write an expression to represent the number of paper plates and plastic forks they bought altogether.

b. There are 100 paper plates in a package and 50 plastic forks in a package. Find how many plates and forks the Johnsons bought.

55. a. Write an expression to find the number of packages needed to buy 375 plastic spoons.

b. If there are 75 plastic spoons in a package, how many packages were bought?

For Exercises 56 and 57, use the information below.
Luz is making a tabletop design with tiles. For each step in his pattern he can determine the number of tiles he needs by multiplying the step number by 6 and subtracting 2.

56. How many tiles will he need for the sixteenth step?

57. In which step will Luz need to use exactly 70 tiles?
For Exercises 58–61, use the figures shown below.

58. Copy and complete the table to show how many tiles are in each figure.

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tiles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

59. Let $s$ be the stage number. Write a rule for finding the number of tiles needed for any stage in the pattern.

60. How many tiles are needed for the ninth stage?

61. If you have 100 tiles, what is the largest stage you can complete?

For Exercises 62–64, use the Input-Output table below.

<table>
<thead>
<tr>
<th>Input ($x$)</th>
<th>16</th>
<th>24</th>
<th>40</th>
<th>52</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output ($y$)</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>11</td>
</tr>
</tbody>
</table>

62. Write a rule that can be used to find $y$ if $x$ is given.

63. If you know $x = 100$, how can you find $y$? Give the value.

64. If you know $y = 20$, how can you find $x$? Give the value.

65. **Multiple Choice** Matt earns $10 for mowing his neighbor’s lawn and $5 an hour for cleaning out the garage. The equation $e = 10 + 5h$ can be used to find his earnings. If he earned $40, how many hours did it take him to clean out the garage?

   A. 4  
   B. 6  
   C. 10  
   D. 210

66. A field is twice as long as it is wide. Let $w$ represent the field’s width.

   a. Write an expression to represent the length of the field.
   b. The field is 40 meters wide. What is the field’s length?
For Exercises 67 and 68, find the value of $x$.

67. \[7.2 + x = 14.1\]

68. \[\frac{3}{10} = x + \frac{1}{5}\]

69. Five friends ate lunch at a restaurant. They had a coupon for $20 off their total bill. The group’s total came to $20 after they used the coupon.

\begin{enumerate}
\item Each person’s lunch had the same price. Write an equation that can be used to determine the price of one lunch.
\item Solve the equation.
\end{enumerate}

70. The table shows the relationship between the number of melons bought and the total cost.

<table>
<thead>
<tr>
<th>Number Bought</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$25</td>
</tr>
<tr>
<td>15</td>
<td>$37.50</td>
</tr>
<tr>
<td>30</td>
<td>$75</td>
</tr>
</tbody>
</table>

\begin{enumerate}
\item Write an expression to find the total cost of buying any number of melons.
\item Sheila is going to buy 62 melons for a banquet. What will be the total cost?
\end{enumerate}

71. Kelly is $x$ years old. Mike is 2 years older than twice Kelly’s age. The sum of Kelly’s and Mike’s ages is 26. How old are Kelly and Mike?

For Exercises 72–78, name the property illustrated in each equation.

72. \[0.85 + (3.5 + 4.15) = (0.85 + 3.5) + 4.15\]

73. \[3d - 15 = 3(d - 5)\]

74. \[0 + (-1.6) + 2.4 = -1.6 + 2.4\]

75. \[\frac{1}{2} \times \frac{2}{1} \times \frac{1}{4} = 1 \times \frac{1}{4}\]

76. \[15(2c - 8) = 30c - 120\]

77. \[-3.2 + (-8.5x) = -8.5x + (-3.2)\]

78. \[123 + (-43) + 0 + (-15) = 123 + (-43) + (-15)\]
79. A carpenter cuts lengths of wood into equal 3-ft. sections. Write an expression to represent the total length of wood the carpenter needs to make \( n \) sections. Evaluate the expression for \( n = 7, 10, \) and 15.

For Exercises 80–82, simplify each expression. Use a property or operation to justify each step.

80. \(-4 + \frac{5}{2} + \frac{6}{3} + \frac{7}{2} + \frac{4}{5}\)

81. \(5m + 6 + 3(m + 2)\)

82. \(-2\left(\frac{1}{2}k + \frac{1}{3}\right) + 6 + \frac{2}{3}\)

83. Copy and complete the table for the given \( x \)-values.

<table>
<thead>
<tr>
<th>( x )-value</th>
<th>( 3(x + 4) )</th>
<th>( 3x + 4 )</th>
<th>( 3x + 12 )</th>
<th>( 2(x + 6) + x )</th>
<th>( 5x + (2x + 12) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Which of the expressions in the top row are equivalent?

84. Multiple Choice Which expression is not equivalent to the others?

A. \(6(x - 2)\)

B. \(2(x - 6) + 4x\)

C. \(6x - 12\)

D. \(7x - (2x + 12)\)

For Exercises 85–88, find the equivalent expression from the box at the right.

85. \(c + c + c\)  
   a. \(c\)

86. \(4c - 2 - 3c + 16\)  
   b. \(3c\)

87. \(c + c + c + c + 2 + 4\)  
   c. \(c + 14\)

88. \(3c + 6c - 8c\)  
   d. \(4c + 6\)
89. Pat earns $9 per hour working as a lifeguard.
   a. Write an algebraic equation to represent the relationship between
      the number of hours Pat works, and the amount that she earns.
   b. Identify the dependent and independent variables in the equation.
   c. Use your equation to complete the table to show the money earned
      for working during the week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours Worked</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Earnings (in $)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For Exercises 90–93, write an algebraic equation to relate the quantities.

90. Michael rides his bike at an average speed of 18 mi/hr for \( t \) hours. He
    travels a total distance of \( d \) miles.

91. Susan is \( y \) years old. She is 8 years younger than her brother, who is
    \( x \) years old.

92. The circumference, \( c \), of any circle is \( 2\pi \) times its radius, \( r \).

93. An object’s mass in kilograms, \( w \), is its mass in grams, \( z \), divided by 1,000.
94. **Multiple Choice**  The solutions to which equation are shown on this graph?

![Graph](image_url)

A. \( g = \frac{p}{2} \)

B. \( p = \frac{g}{2} \)

C. \( p = g - 2 \)

D. \( g = p + 2 \)

95. The table shows the prices of some produce at a farmers’ market.

<table>
<thead>
<tr>
<th>Produce</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apples</td>
<td>$7 per basket</td>
</tr>
<tr>
<td>Pears</td>
<td>$7 per basket</td>
</tr>
<tr>
<td>Corn</td>
<td>$0.75 per ear</td>
</tr>
<tr>
<td>Asparagus</td>
<td>$3.50 per bundle</td>
</tr>
<tr>
<td>Broccoli</td>
<td>$2.50 per bag</td>
</tr>
</tbody>
</table>

Write two equivalent algebraic expressions to represent the total cost.

a. some ears of corn

b. 3 baskets of apples and some baskets of pears

c. some baskets of apples, 2 baskets of pears, and 4 ears of corn