CC Investigation 2: Equivalent Expressions

Mathematical Goals

• Apply the properties of operations to add, subtract, factor, and expand algebraic expressions.

• Understand that writing an equivalent expression in a problem context can shed light on how quantities in the problem are related.

Teaching Notes

In order for students to understand and appreciate the purpose of finding equivalent expressions, they first must understand that algebraic expressions can be written to represent problem situations. Give students practice in writing simple algebraic expressions by having them represent these situations:

• 3 more than a number
• 8 degrees less than yesterday’s temperature
• some boxes of pencils with 25 pencils in each box
• splitting the cost of dinner equally among 4 friends

Have students explain what each value in their expressions represents.

Before beginning the problems, review the associative, commutative, and distributive properties, using different types of rational numbers.

**Associative Property**

\[ 4 + (3 + 8) = (4 + 3) + 8 \]

\[ (3 \times 7.5) \times 2 = 3 \times (7.5 \times 2) \]

**Commutative Property**

\[ \frac{1}{4} + \frac{3}{8} = \frac{3}{8} + \frac{1}{4} \]

\[ 4 \times \frac{5}{6} = \frac{5}{6} \times 4 \]

**Distributive Property**

\[ 6 \times (8.5 + 9) = (6 \times 8.5) + (6 \times 9) \]
Problem 2.1

During Problem 2.1A, ask: Why would you want to simplify the expression $25j + 11.5(2 + j)$? (to make it easier to find the value of the expression when given a value for $j$)

During Problem 2.1A, guide students through the steps of the simplification. Ask:

- How is the second expression different than the first? (The coefficient 11.5 has been multiplied separately by each addend inside the parentheses.)
- What property does that demonstrate? (distributive property)
- How is the third expression different than the second? (The order of the addends is changed.)
- What property does that demonstrate? (commutative property)
- How is the fourth expression different than the third? (The coefficients of $j$ have been placed together inside parentheses.)
- What property does that demonstrate? (distributive property)

Before Problem 2.1A, Part 4, explain that to evaluate an expression, students should substitute the given value for $j$ and then simplify.

After Problem 2.1, ask: Was it easier to evaluate the expression in Part 4 or in Part 5? Why? (Part 5 was easier to evaluate because there were fewer terms.)

Problem 2.2

Before 2.2A, review with students how to express a percent as an equivalent decimal. Ask: What decimal is equivalent to 20%? (0.2)

During Problem 2.2A, ask: What equivalent expression can you write for $p – 0.2p$, using the distributive property? ($p(1 – 0.2)$)

During Problem 2.2A, Part 4, ask:

- What expression can you write to represent the sale price of an item that is 25% off? ($p – 0.25p$)
- What equivalent expression can you write using the distributive property? ($p(1 – 0.25)$)
- What expression can you write to represent the price of an item that is on sale for 75% of its original price? ($0.75p$)

During Problem 2.2C, ask: How can you tell from looking at the expression which term represents the discount and which represents the tax? (The term being subtracted represents a discount off the original price, while the term being added represents the tax added to the price.)

Summarize

To summarize the lesson, ask:

- When might you use a property of operations to write an equivalent expression? (to simplify an expression so that you can evaluate it using mental math)
- What property is used to rewrite the expression $3.6a + 4.4a$ as $(3.6 + 4.4)a$? (distributive property)
- What is another way to express 0.7 times a number? (the number minus 0.3 times the number)
Assignment Guide for Investigation 2
Problem 2.1, Exercises 1–17
Problem 2.2, Exercises 18–23

Answers to Investigation 2

Problem 2.1
A. 1. the cost of the pairs of jeans Chris buys
    2. 11.5(2 + j)
    3. 

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>25j + 11.5(2 + j)</td>
<td>original expression</td>
</tr>
<tr>
<td>25j + 11.5(2) + 11.5(j)</td>
<td>distributive property</td>
</tr>
<tr>
<td>25j + 11.5(j) + 11.5(2)</td>
<td>commutative property of addition</td>
</tr>
<tr>
<td>(25 + 11.5)j + 11.5(2)</td>
<td>distributive property</td>
</tr>
<tr>
<td>(36.5)j + 11.5(2)</td>
<td>addition</td>
</tr>
<tr>
<td>36.5j + 23</td>
<td>multiplication</td>
</tr>
</tbody>
</table>

4. For 2 pairs of jeans: 25j + 11.5(2 + j) = 25(2) + 11.5(2 + 2) = 50 + 11.5(4) = 50 + 46 = $96; for 4 pairs of jeans: 25j + 11.5(2 + j) = 25(4) + 11.5(2 + 4) = 100 + 11.5(6) = 100 + 69 = $169.

5. For 2 pairs of jeans: 36.5j + 23 = 36.5(2) + 23 = 73 + 23 = $96; for 4 pairs of jeans: 36.5j + 23 = 36.5(4) + 23 = 146 + 23 = $169; the expressions have the same values for a given value of j, so the expressions are equivalent.

B. 1. 78 represents the money Chris has; 20t represents the cost of t dress shirts; \( \frac{1}{2}(12t) \) is the cost of t ties at \( \frac{1}{2} \) off.

Problem 2.2
A. 1. \( p \) represents the full price of the item; 0.2\( p \) represents the 20% discount.
    2. The sale price is the full price, \( p \), minus the 20% discount, or \( p - 0.2p \).
    3. Yes, according to the distributive property, \( p - 0.2p = (1 - 0.2)p = 0.8p \).
    4. The sales are equivalent. A sale of 25% off gives a sale price of 100% – 25% = 75%.

B. 1. \( c \) represents the price of an item; 0.06\( c \) represents the 6% sales tax on the item.
    2. The price, including tax, is the item’s price, \( c \), plus the tax, 0.06\( c \), or \( c + 0.06c \).
    3. Factor \( c \) out of both terms, and then add the remaining terms: \( c + 0.06c = (1 + 0.06)c = 1.06c \).
    4. The expressions are equivalent: \( c + 0.06c = 1.06c \).
5. \(c + 0.06c = 20 + (0.06)20 = 20 + 1.2 = \$21.20\); 1.06c = 1.06(20) = \$21.20; The expressions have the same value because they are equivalent.

6. The cost of the jacket with tax is 1.06c = 1.06(20) = \$21.20; The expressions have the same value because they are equivalent.

C. 1. 15\%; the term – 0.15\(d\) represents a savings of 15\% of the original price, \(d\).
C. 2. 7\%; the term + 0.07\(d\) represents a tax of 7\% on the original price, \(d\).
C. 3. \(d – 0.15d + 0.07d = (1 – 0.15 + 0.07)d = 0.92d\)

Exercises

1. 

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(p + 7) – 2p</td>
<td>original expression</td>
</tr>
<tr>
<td>4p + 28 – 2p</td>
<td>distributive property</td>
</tr>
<tr>
<td>4p – 2p + 28</td>
<td>commutative property</td>
</tr>
<tr>
<td>(4 – 2)p + 28</td>
<td>distributive property</td>
</tr>
<tr>
<td>2p + 28</td>
<td>subtraction</td>
</tr>
</tbody>
</table>

2. 10t; distributive property
3. \(\frac{5}{3}x\); distributive property
4. 15m – 55; distributive property
5. 4p – 12; distributive property, commutative property
6. 1.2n + 8.4; distributive property, commutative property
7. –2g + 4h – 22; distributive property, commutative property, associative property
8. 4(x – 3); 4(–3 + x)
9. a. \(2m + 20\), where \(m\) represents the amount of money Marty has.
   b. \(m + 2m + 20 = (1 + 2)m + 20 = 3m + 20\); distributive property
   c. \(\frac{1}{2}(3m + 20) = \frac{3}{2}m + 10\); distributive property
   d. Aimee: \(2m + 20 = 2(30) + 20 = \$80\);
      Jack: \(\frac{3}{2}m + 10 = \frac{3}{2}(30) + 10 = 45 + 10 = \$55\)
10. No, \(x + 8 + 3x + 12y = 4x + 8 + 12y\);
    4(x + 8) + 12y = 4x + 32 + 12y;
    4x + 8 + 12y \neq 4x + 32 + 12y.
11. first week: 8; second week: \(h\); third week: 2\(h\);
    total: \(8 + h + 2h = 8 + 3h\)
12. D
13. B
14. F
15. E
16. A
17. C
18. a. \(w + 0.07w\)
   b. \(1.07w\)
19. \(d + 0.04d; 1.04d\)
20. \(x – 0.25x; 0.75x\)
21. \(c + 0.75c; 1.75c\)
22. Yes, \(0.83p = (1 – 0.17)p = p – 0.17p\).
23. Yes, \(y – \frac{1}{5}y = \left(1 – \frac{1}{5}\right)y = \frac{4}{5}y\).
Marcus is shopping at Sound World. Prices are listed in the table.

<table>
<thead>
<tr>
<th>Sound World</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
</tr>
<tr>
<td>DVD</td>
</tr>
<tr>
<td>CD</td>
</tr>
<tr>
<td>Video Game</td>
</tr>
<tr>
<td>Computer Game</td>
</tr>
</tbody>
</table>

1. Marcus buys some DVDs. He buys 3 more CDs than he buys DVDs. He writes the expression $24d + 11(3 + d)$ to represent the total cost.
   a. Describe what each part of Marcus’ expression represents.
   b. Copy and complete the table to show the steps to simplifying the expression.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24d + 11(3 + d)$</td>
<td>original expression</td>
</tr>
<tr>
<td>$24d + 11(3) + 11(d)$</td>
<td></td>
</tr>
<tr>
<td>$24d + 11(d) + 11(3)$</td>
<td></td>
</tr>
<tr>
<td>$(24 + 11)d + 11(3)$</td>
<td></td>
</tr>
<tr>
<td>$(35)d + 11(3)$</td>
<td></td>
</tr>
<tr>
<td>$35d + 33$</td>
<td></td>
</tr>
</tbody>
</table>

c. Evaluate the original expression, $24d + 11(3 + d)$, to find the total cost if Marcus buys 3 DVDs or 5 DVDs.

d. Evaluate the simplified expression, $35d + 33$, for the same values of $d$. Compare the answers to what you found. What does this tell you about the expressions?

2. a. Marcus can save 15% on his purchase by signing up for a Sound World credit card. Write two equivalent expressions to show the discounted price of an item with an original price of $p$.
   b. Marcus writes the expression $p + 0.06p$ to represent the price of an item including sales tax. Tell what tax rate the store charges, and write an equivalent expression to represent the total price.
Skill: Evaluate Expressions

For Exercises 1–6, evaluate the expression for \( p = 4 \).

1. \( 3p \)
2. \( 8p + 9 \)
3. \( 16 \div p \)
4. \( p(4 + p) \)
5. \( 6 - \frac{p}{2} \)
6. \( \frac{p + 24}{7} \)

For Exercises 7–12, evaluate the expression for \( g = 0.7 \).

7. \( 1 - g \)
8. \( 4(g + 7) \)
9. \( 13g \)
10. \( g + 0.8 + g \)
11. \( 45g + g \)
12. \( 12g + 12 \)

Skill: Simplify Expressions

Simplify the expression.

13. \( 6y - (4 + y) \)
14. \( 2.5(4g - 2) + 1.8g \)
15. \( \frac{2}{5}r + \frac{4}{5}r \)
16. \( -(t + 4) - 7(t + 8) \)
17. \( 8(14 - 6f) \)
18. \( 2d + 13d - 6d \)
19. \( 9(a - 6) + 6(9 - a) \)
20. \( \frac{1}{2}(u - 6) + \frac{u}{2} \)
21. \( 9q + \frac{3}{2}(6q + 4) \)
22. \( 4(4.1 - 2v) + 3v \)
1. At the start of the season, a baseball team buys some balls for each player on the team. It also buys 4 fewer bats than balls for each player. The expression $12b + 12(b - 4)$ represents the total number of bats and balls the team buys for 12 players.
   
   a. Describe what each part of the expression represents.
   
   b. Complete the table to show the steps to simplifying the expression.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12b + 12(b - 4)$</td>
<td>original expression</td>
</tr>
<tr>
<td>$12[b + (b - 4)]$</td>
<td></td>
</tr>
<tr>
<td>$12[(b + b) - 4]$</td>
<td></td>
</tr>
<tr>
<td>$12(2b - 4)$</td>
<td></td>
</tr>
<tr>
<td>$12(2b) - 12(4)$</td>
<td></td>
</tr>
<tr>
<td>$24b - 48$</td>
<td></td>
</tr>
</tbody>
</table>

   c. Evaluate the original expression, $12b + 12(b - 4)$, to find the total number of bats and balls if the team buys 6 balls for each player. Show your work.

   d. Evaluate the simplified expression, $24b - 48$, for the same value of $b$. Show your work. Compare the answer to the answer in Part c. What does this tell you about the expressions?

2. SportsTown is having a 30%‐off sale on baseball equipment. Andy finds the sale price of a baseball glove that normally costs $49 by evaluating the expression $49 - 0.3(49)$. Juan finds the sale price by multiplying $0.7(49)$. Who found the correct sale price? Explain how you know.
3. The table shows the prices of two stocks.

<table>
<thead>
<tr>
<th>Stock Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
</tr>
<tr>
<td>Fine Motors, Inc.</td>
</tr>
<tr>
<td>AutoMart Corp.</td>
</tr>
</tbody>
</table>

a. Fine Motors’ stock gains 5% per share. Write two equivalent expressions to represent the stock’s new price. Evaluate each expression to find the new price. Show your work.

b. Over one month, Fine Motors’ stock gains 12% per share over the price shown in the table. What percent gain would AutoMart’s stock need to have over the same period for the two stocks to have the same price? Explain how you know.

4. Lisa buys a computer on sale for 25% off. The store adds 6% sales tax to the price of the computer after the discount.

a. The expression $0.75c + 0.06(c – 0.25c)$ represents the final price, including tax, of a computer with a regular price of $c$ dollars. Simplify the expression. List the properties of operations you use.

b. How much does Lisa pay for a computer with a regular price of $700? Show your work.
Investigation 2: Equivalent Expressions

You can use the properties of operations, such as the associative, commutative, and distributive properties, to simplify algebraic expressions.

Problem 2.1

A. Chris buys some pairs of jeans. He buys 2 more pairs of shorts than he buys jeans. He writes the expression $25j + 11.5(2 + j)$ to represent the total cost.

1. What does the term $25j$ in the expression mean?
2. What part of the expression represents the cost of the shorts Chris buys?
3. Copy and complete the table to show the steps to simplifying the expression.

<table>
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<tr>
<td>$36.5j + 23$</td>
<td></td>
</tr>
</tbody>
</table>

4. Evaluate the original expression, $25j + 11.5(2 + j)$, to find the total cost if Chris buys 2 pairs of jeans or 4 pairs of jeans.
5. Evaluate the simplified expression, $36.5j + 23$, for the same values of $j$. What does this tell you about the expressions?

B. Chris has $78 and a coupon that lets him buy one tie for $\frac{1}{2}$ off for each dress shirt he buys. He writes the expression $78 - 20t - \frac{1}{2}(12t)$ to represent how much money he will have left if he buys $t$ shirts and ties.

1. Explain what each term in the expression represents.
2. Simplify the expression. Give a reason for each step.
3. Evaluate each expression for $t = 3$. Does each expression have the same value? Explain why or why not.
A. The store where Chris is shopping is having a 20%-off sale. Chris writes the expression \( p - 0.2p \) to represent the sale price of any item.

1. What do the terms \( p \) and \( 0.2p \) in the expression represent?
2. Explain how the expression \( p - 0.2p \) represents the sale price of an item.
3. Does the expression \( 0.8p \) also represent the sale price of an item? Explain why or why not.
4. Two other stores also are having sales. One store offers 25% off, and the second store offers its items at 75% of their original prices. Which sale is better? Explain your choice.

B. A 6% sales tax is added to all purchases at the store. Chris writes the expression \( c + 0.06c \) to represent the cost of an item, including tax.

1. What do the terms \( c \) and \( 0.06c \) in the expression represent?
2. Explain how the expression \( c + 0.06c \) represents the cost of an item, including tax.
3. Describe how to use the distributive property to simplify the expression \( c + 0.06c \). Write the simplified expression.
4. Explain why both expressions represent the same situation.
5. Evaluate each expression for \( c = $20 \). Do the expressions have the same value? Explain why or why not.
6. Chris wants to buy a jacket that costs $225 before the 6% tax. He can buy the same jacket online for $235 with no sales tax. Where should Chris buy the jacket? Explain your choice.

C. Chris buys a pair of shoes with an original price of \( d \) dollars. He writes the expression \( d - 0.15d + 0.07d \) to represent the final cost of the shoes.

1. The shoes are on sale. What percent off is the sale? Explain how you know.
2. Chris includes sales tax in the final cost of the shoes. What sales tax rate did he use? Explain how you know.
3. Simplify the expression. Show your work.
Exercises
1. Copy and complete the table by providing the property or reason for each step.

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<td>4p + 28 – 2p</td>
<td></td>
</tr>
<tr>
<td>4p – 2p + 28</td>
<td></td>
</tr>
<tr>
<td>(4 – 2)p + 28</td>
<td></td>
</tr>
<tr>
<td>2p + 28</td>
<td></td>
</tr>
</tbody>
</table>

For Exercises 2–7, simplify the expression. List the properties you used.
2. 3t – 7t + 14t
   - 3x + \( \frac{4}{3}x \)
3. \( \frac{1}{3}x + \frac{4}{3}x \)
4. 5(3m – 11)
5. 5p – (12 + p)
6. 1.2(3n + 7) – 2.4n
7. –2(g + 5) + 4(h – 3)
8. Write an expression that can be simplified to 4x – 12 using the distributive property. Write another equivalent expression using the commutative property.
9. Marty has some money. Aimee has $20 more than twice as much money as Marty has. Jack has one half of the sum of the amounts that Marty and Aimee have.
   a. Write an expression for the amount of money Aimee has.
   b. Write and simplify an expression for the sum of the amounts of money that Aimee and Marty have. Which property or properties did you use to simplify the expression?
   c. Write and simplify an expression for the amount of money Jack has. Which property or properties did you use to simplify the expression?
   d. If Marty has $30, how much money do Aimee and Jack each have?
10. James says the expressions \( x + 8 + 3x + 12y \) and \( 4(x + 8) + 12y \) are equivalent. Is James correct? Explain your answer.
11. You worked a total of 8 hours the first week of this month. The second week you worked less, but you can’t remember how many hours less. You know you worked twice as many hours in the third week as in the second week. Write and simplify an expression for the number of hours you worked the first three weeks of this month. Show your steps.
For Exercises 12–17, match the expression with an equivalent expression from the box at the right.

12. $x - 0.7x$   A. $1.3x$
13. $1.5x$       B. $x + 0.5x$
14. $2x - 2$     C. $2x + 2$
15. $x - 0.5x$   D. $0.3x$
16. $x + 0.3x$   E. $0.5x$
17. $2(x + 1)$   F. $2(x - 1)$

18. A 7% sales tax is added to all purchases at an electronics store. Carmella wants to buy a digital music player that costs $w$ dollars before tax.
   a. Write an expression using addition that shows the total cost of the music player, including tax.
   b. Write an equivalent expression using only multiplication, that also shows the total cost of the music player, including tax.

For Exercises 19–21, write two equivalent expressions you could use to solve the problem.

19. A new employee earns $d$ dollars per year. She gets a raise of 4% after her first year. What is her new salary?

20. You are buying some CDs. You have a coupon for 25% off of your purchase. How much do you spend if your purchase before the discount is taken is $x$ dollars?

21. The number of cells, $c$, in a Petri dish increases by 75% in the first hour. How many cells are in the dish after the first hour?

22. The population of a city decreased this year by 17% from the population, $p$, last year. A resident of the city says that you can find the city’s population this year by finding $0.83p$. Is the resident correct? Explain your answer.

23. Can the expression $y - \frac{1}{5}y$ be used to represent the same situation as represented by the expression $\frac{4}{5}y$? Explain why or why not.